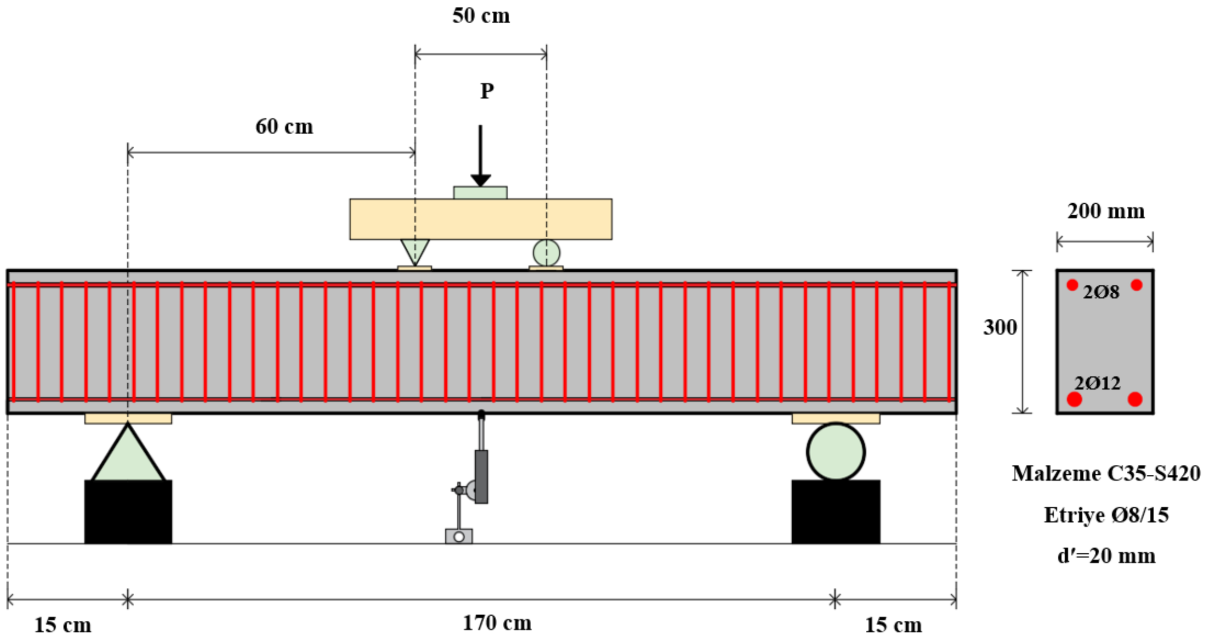
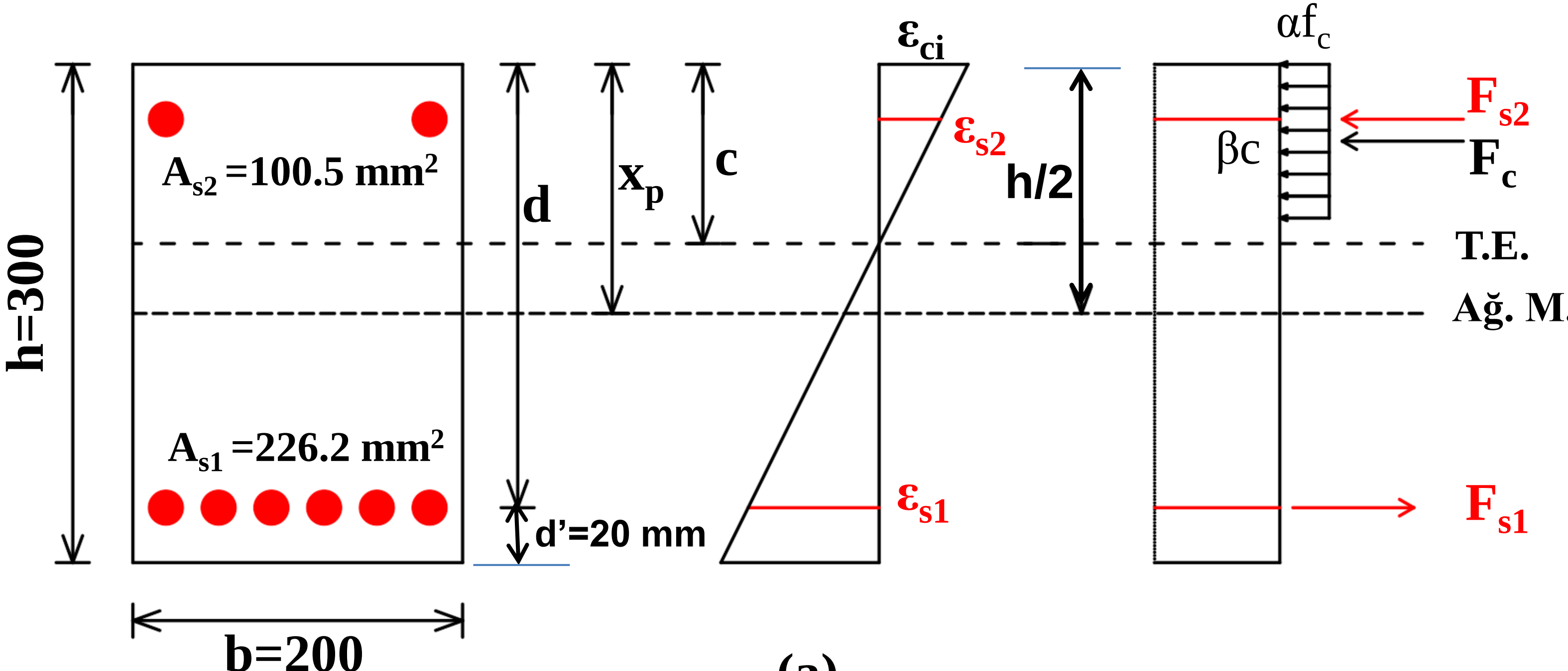
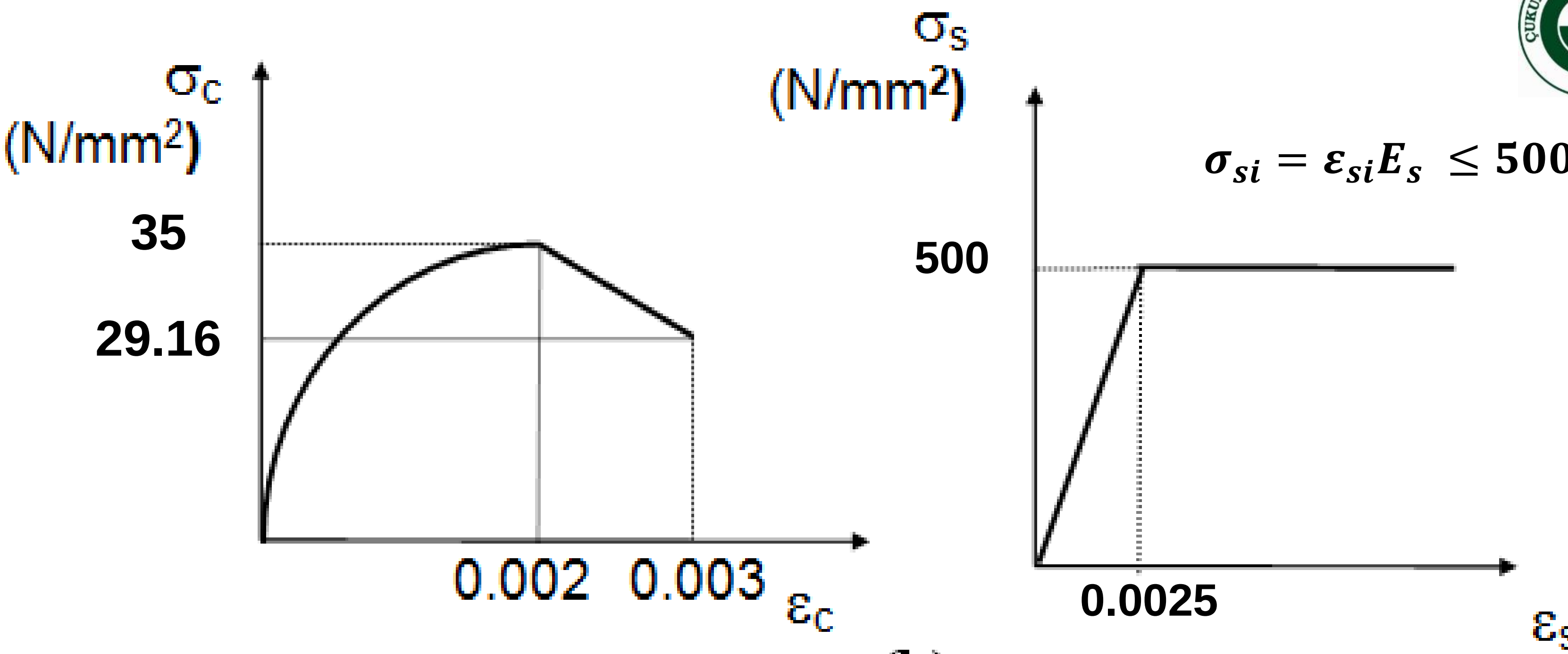

50 cm
P
60 cm
200 mm
300
2Ø8
2Ø12
Malzeme C35-S420
Etriye Ø8/15
d′=20 mm
15 cm
170 cm
15 cm


h=300
A_{s2} =100.5 mm^2
A_{s1} =226.2 mm^2
b=200
d
x_p
c
ε_{ci}
ε_{s2}
h/2
ε_{s1}
d'=20 mm
αf_c
F_{s2}
βc
F_c
T.E.
Ağ. M.
F_{s1}

(a)

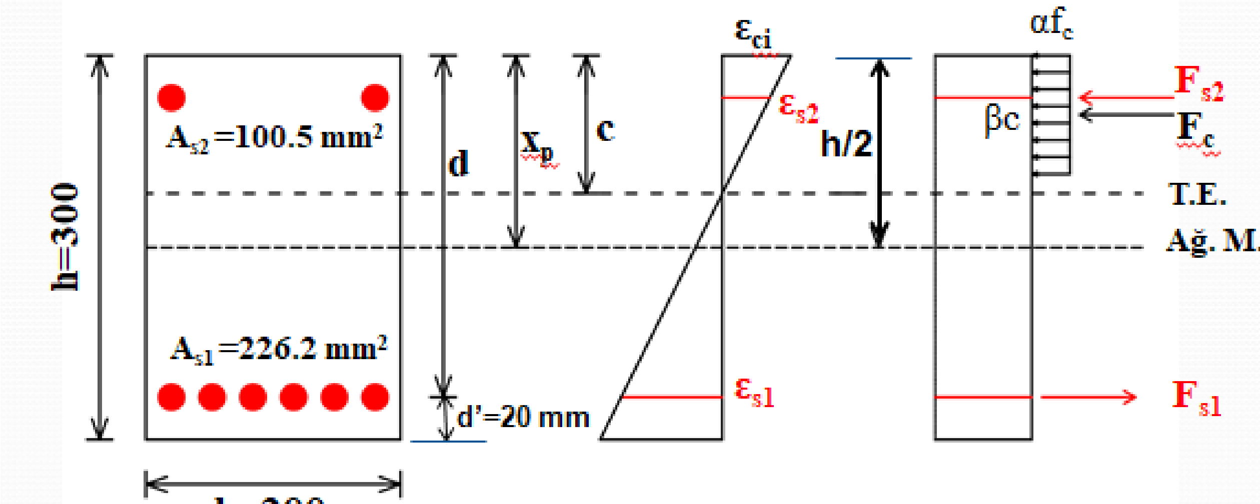


Beton (b) Çelik

Çözüm:

Moment eğrilik ilişkisini gösteren eğri, yeterli sayıda noktanın bulunması ve bunların birleştirilmesinden elde edilir. Noktaların koordinatları M_i ve K_i ile belirlenir.

$$\varepsilon_{ci} = 0.002 \;\rightarrow\; c = ?\;\; M_i = ?\;\; K_i = ?$$



(a)

$$N = F_c + F_{s2} - F_{s1}$$

$$\varepsilon_{s1} = \left(\frac{d-c}{c}\right) * 0.002$$

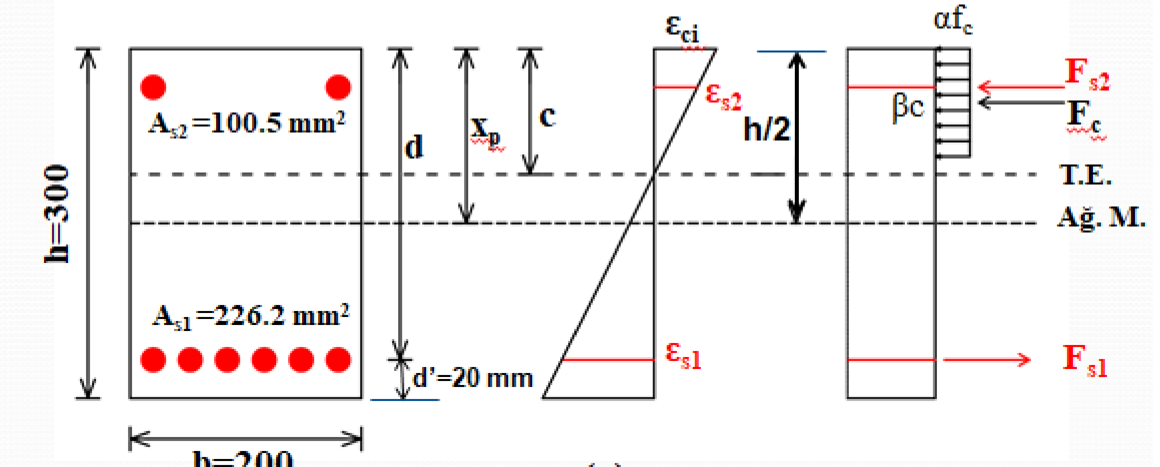
$$\sigma_{s1} = \left(\frac{d-c}{c}\right) 0.002 \; *2*10^5 \leq 500\ Mpa \quad \varepsilon_{s1} \geq \varepsilon_{sy} \quad \sigma_{s1} = 500\ Mpa$$

$$\varepsilon_{s2} = \left(\frac{c-d'}{c}\right) * 0.002$$

$$\sigma_{s2} = \left(\frac{c-d'}{c}\right) 0.002 \; *2*10^5 = 400\left(\frac{c-d'}{c}\right)$$

$$0 = \alpha\beta f_c\,(c)b + 100.5 \times 400\left(\frac{c-d'}{c}\right) - 226.2 \times 400\left(\frac{d-c}{c}\right)$$

$$0 = 0.889 \times 0.750 \times 35\,(c) \times 200 + 100.5 \times 400\left(\frac{c-20}{c}\right) - 226.2 \times 500$$



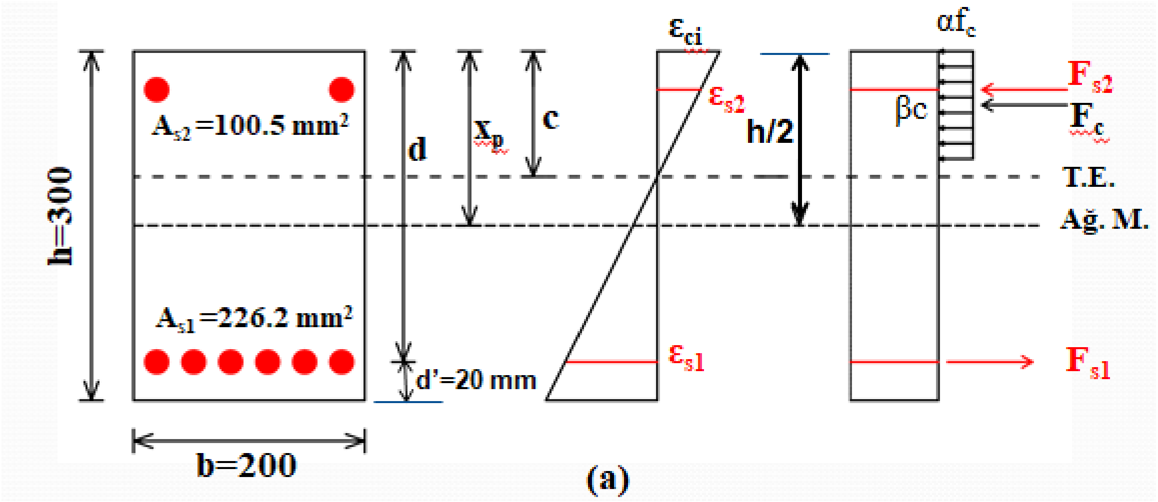
(a)

$$0 = 0.889 \times 0.750 \times 35\,(c) \times 200 + 100.5 \times 400\left(\frac{c-20}{c}\right) - 226.2 \times 500$$

$$4667.25c^2 + 40200c - 804000 - 113100c = 0$$

$$4667c^2 - 72900c - 804000 = 0$$

$$c = 23.08\,mm \quad \sigma_{s2} = 400\left(\frac{23.08-20}{23.08}\right) = 53.37\,MPa$$



$$F_c = 0.889 \times 0.750 \times 35 \times 23.08 \times 200 \times 10^{-3} = 107.72\ kN$$

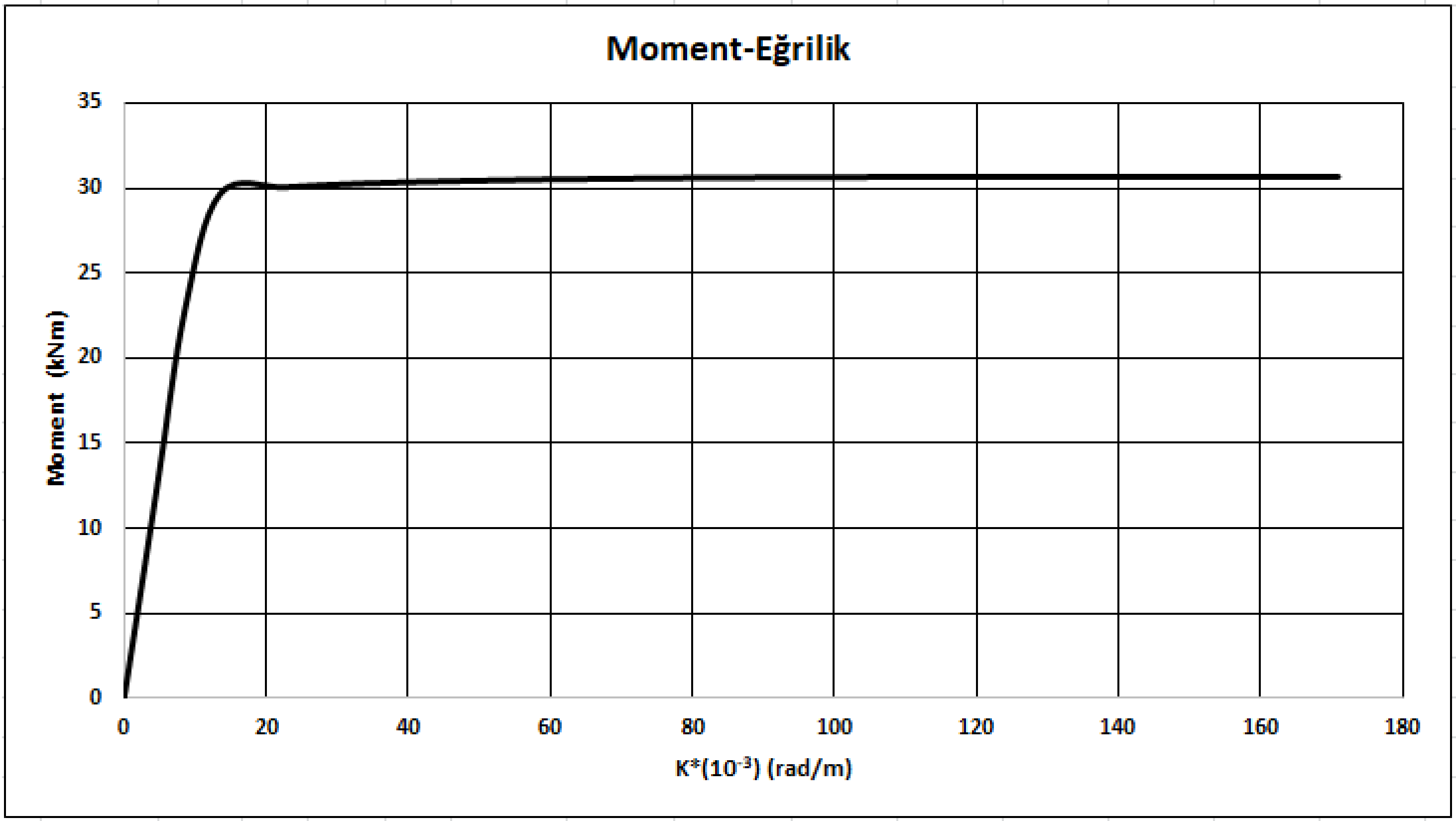
$$F_{s1} = -226.2 \times 500 \times 10^{-3} = -113.1\ kN$$

$$F_{s2} = 101 \times 53.37 \times 10^{-3} = 5.39\ kN$$

$$M_i = 107.72\left(150 - \frac{0.92 \times 23.08}{2}\right) + 5.39 \times (150 - 20)$$
$$-113.1 \times \{-(150 - 20)\} = 30418\ kNmm = 30.418\ kNm$$

$$K_i = \frac{0.002}{0.02308} = 0.086\ rad/m$$

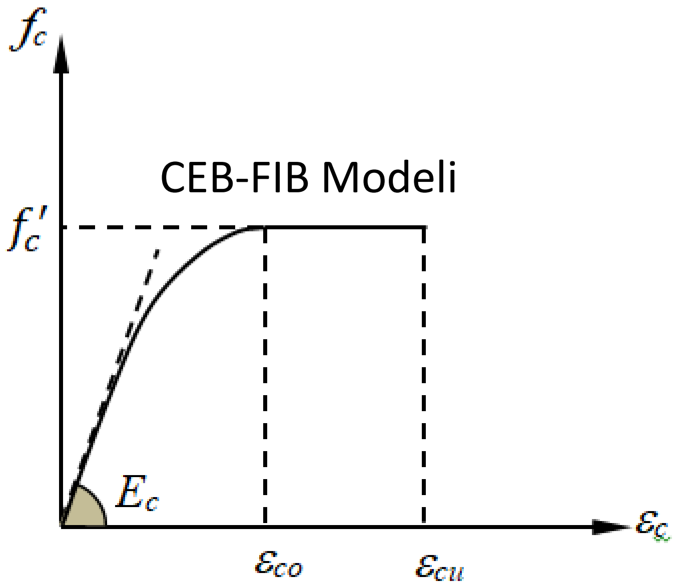
$\varepsilon_c *10^{-3}$	c (mm)	N (kN)	M (kNm)	K $*10^{-3}$
			0	0
0.25	53.40	0.0	12.57	4.68
0.47	54.39	0.0	22.89	8.58
0.68	50.45	0.0	29.68	13.54
0.90	39.35	0.0	30.06	22.87
1.12	33.10	0.0	30.27	33.74
1.33	29.16	0.0	30.41	45.73
1.55	26.34	0.0	30.51	58.84
1.77	24.63	0.0	30.57	71.73
1.98	23.18	0.0	30.62	85.55
2.20	22.41	0.0	30.65	98.16
2.42	21.76	0.0	30.67	111.08
2.63	21.31	0.0	30.68	123.60
2.85	20.98	0.0	30.68	135.85
3.07	20.73	0.0	30.68	147.96
3.28	20.60	0.0	30.67	159.39
3.50	20.48	0.0	30.67	170.90

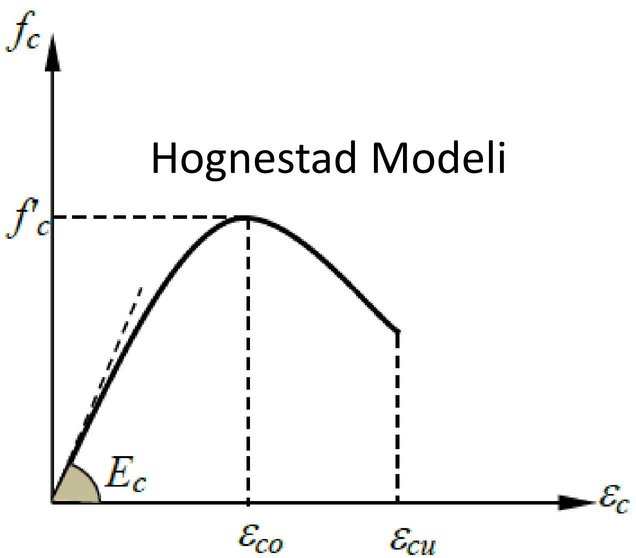

Moment-Eğrilik
Moment (kNm)
35
30
25
20
15
10
5
0
0
20
40
60
80
100
120
140
160
180
K*(10-3) (rad/m)

Lifli Polimer (FRP) ve çelik donatılı betonarme kirişin moment-eğrilik ilişkisinin elde edilmesi

Dundar C., Tanrikulu A. K., Frosch J. R. (2015). "Prediction of load-deflection behavior of multi-span FRP and steel reinforced concrete beams", Composite Structures, 132, 680-693, 2015.

Betonun Basınç Altında Gerilme-Deformasyon Modeli





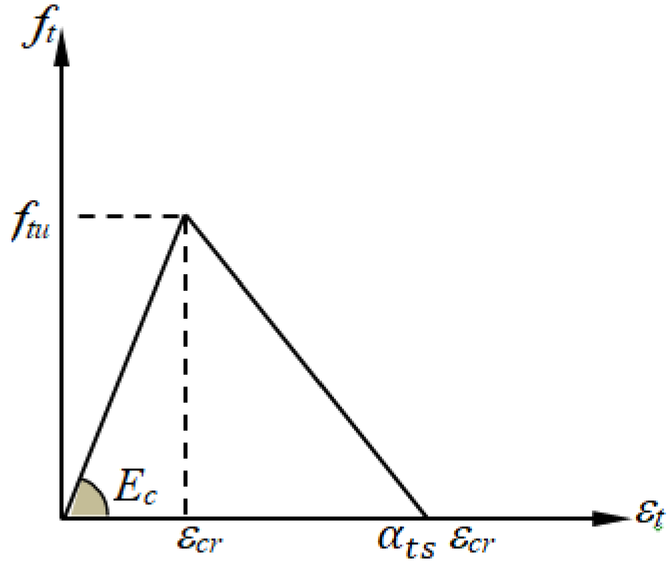
Analiz yönteminde beton için herhangi bir model kullanılabilir. Örneğin, CEB-FIP modeli kullanılıyorsa, aşağıdaki denklemler dikkate alınır:

$$f_c = f_c' \left[\frac{2\varepsilon_c}{\varepsilon_{co}} - \left(\frac{\varepsilon_c}{\varepsilon_{co}} \right)^2 \right] \qquad \varepsilon_c \leq \varepsilon_{co}$$

$$f_c = f_c' \qquad \varepsilon_{co} \leq \varepsilon_c \leq \varepsilon_{cu}$$

Eğer Hognestad modeli kullanılırsa ilk denklemde beton birim kısalması $\varepsilon_c \leq \varepsilon_{cu}$ alınmalıdır. $\varepsilon_{co} = 0.002$ ve $\varepsilon_{cu} = 0.0035$

Betonun Çekme Altında Gerilme-Deformasyon Modeli



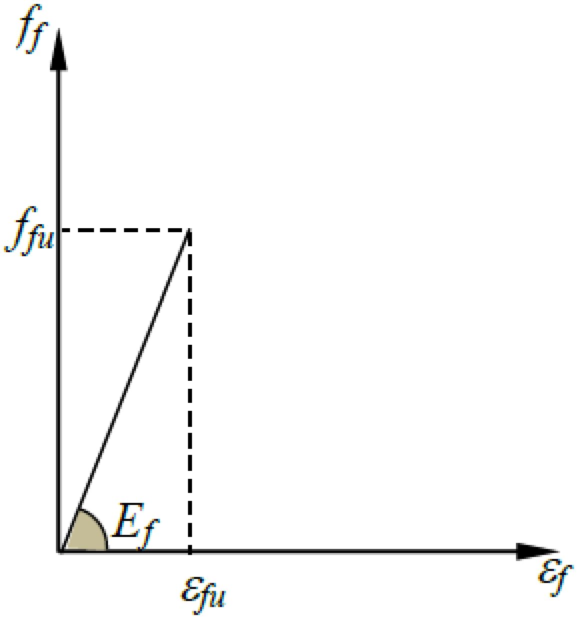
Betonun çekme gerilmesi-birim çekme ilişkisi için herhangi bir modeli de kullanılabilir. Bilineer gerilme-şekil değiştirme ilişkisi kullanılıyorsa, aşağıdaki denklemler dikkate alınır:

$$f_t = E_c \varepsilon_t \qquad \varepsilon_t \le \varepsilon_{cr}$$

$$f_t = f_r - \frac{f_r}{\varepsilon_{ctu} - \varepsilon_{cr}}(\varepsilon_t - \varepsilon_{cr}), \qquad \varepsilon_{ctu} \ge \varepsilon_t \ge \varepsilon_{cr}$$

$$\varepsilon_{ctu} = \alpha_{ts}\, \varepsilon_{cr}$$

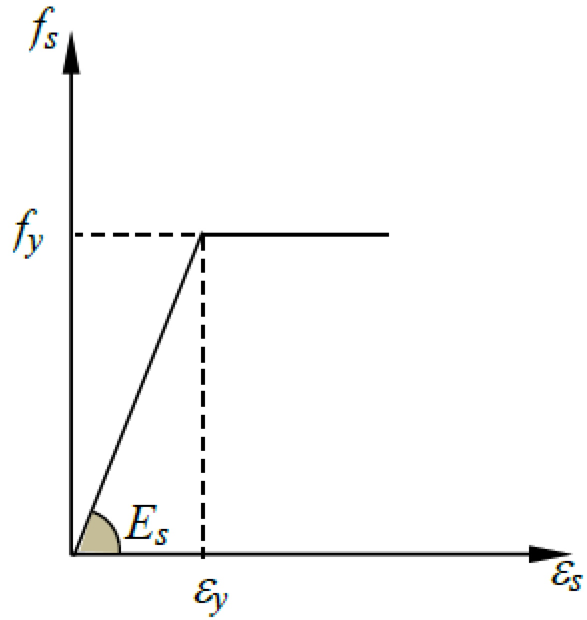
FRP Donatısının Çekme Altında Gerilme-Deformasyon Modeli



FRP çubuklarının gerilme-deformasyon ilişkisi kopmaya kadar doğrusal elastiktir:

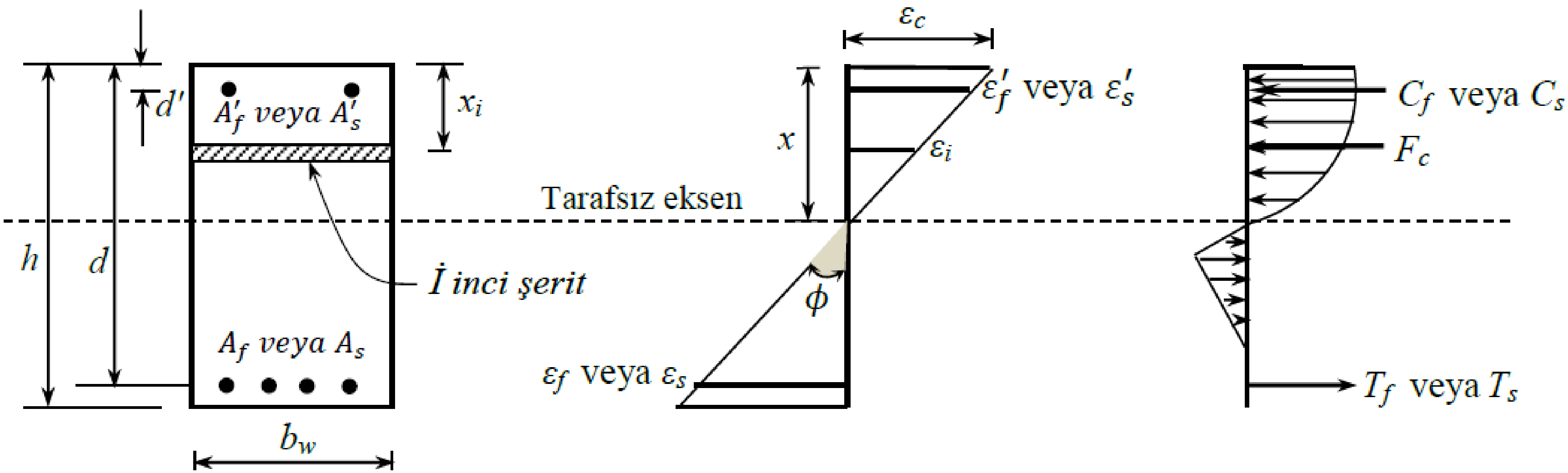
$$f_f = E_f \varepsilon_f \qquad \varepsilon_f \leq \varepsilon_{fu}$$

Çelik Donatının Gerilme-Deformasyon Modeli



Çeliğin gerilme-uzama ilişkisi, elastik-plastik bir malzeme olarak modellenmiştir:

$$f_s = E_s \varepsilon_s \leq f_y$$



(a) Betonarme kesit

(b) Birim deformasyon dağılımı

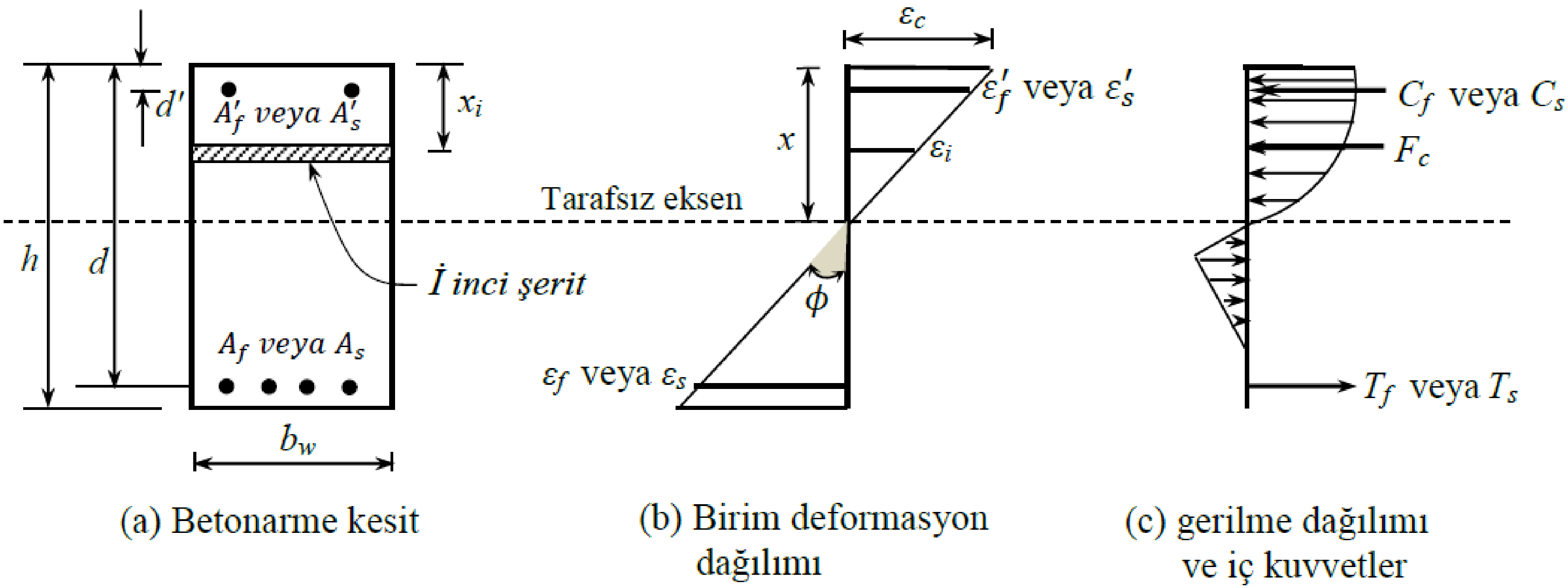
(c) gerilme dağılımı ve iç kuvvetler

FRP veya çelik donatılı bir betonarme kesitte birim deformasyon, gerilmeler ve iç kuvvetler

$$F_c = \sum_{i=1}^{n} f_{ci} h_i b$$

$$C_f = A'_f E_f \varepsilon'_f \qquad C_s = A'_s E_s \varepsilon'_s$$

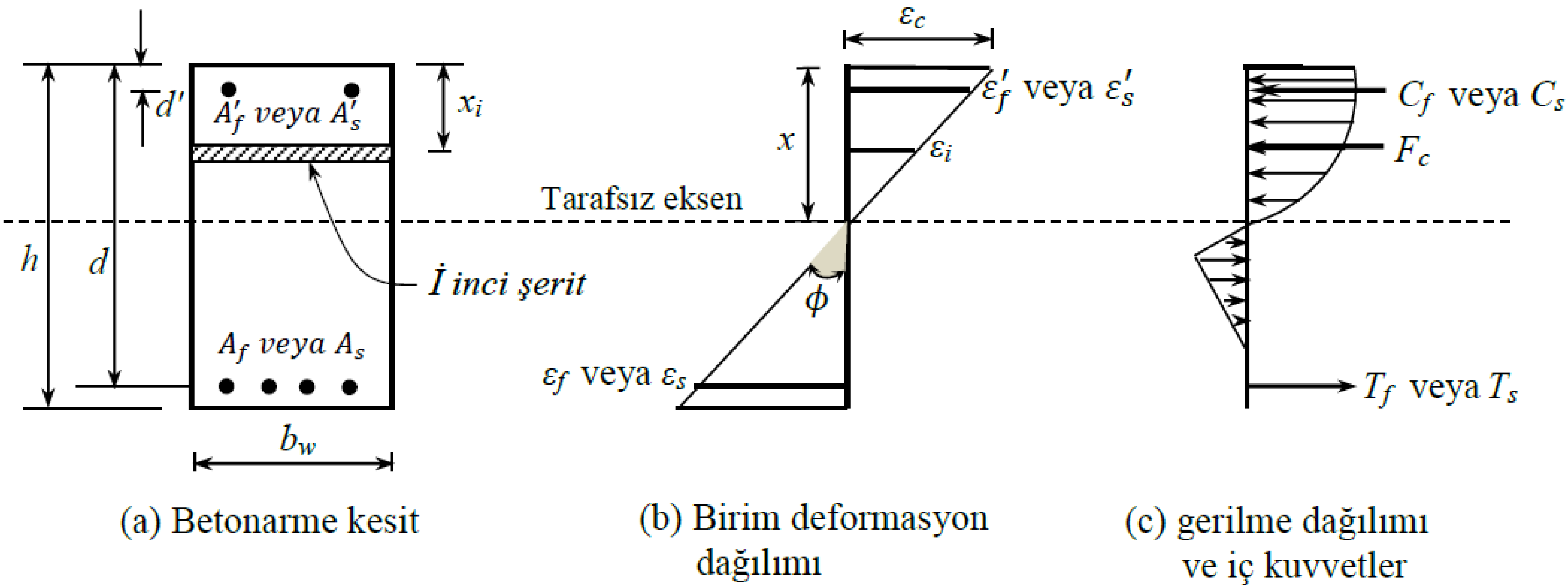
$$T_f = A_f E_f \varepsilon_f \qquad T_s = A_s E_s \varepsilon_s$$



FRP veya çelik donatılı bir betonarme kesitte birim deformasyon, gerilmeler ve iç kuvvetler

Başlangıçta varsayılan tarafsız eksen derinliği, yeterli olana kadar yinelemeli olarak düzeltilir denge doğruluğu, aşağıdaki yakınsama kriteri kullanılarak elde edilir

$$\frac{|\sum F|}{|F_c|} \leq \epsilon \qquad \epsilon = 10^{-8}$$

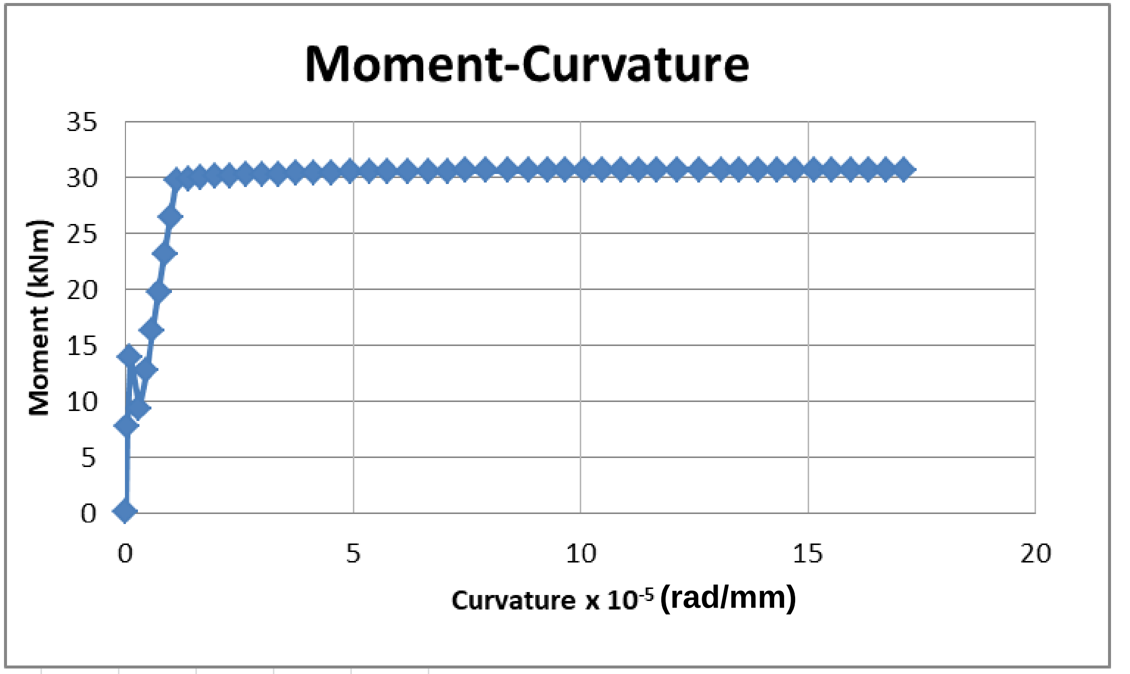


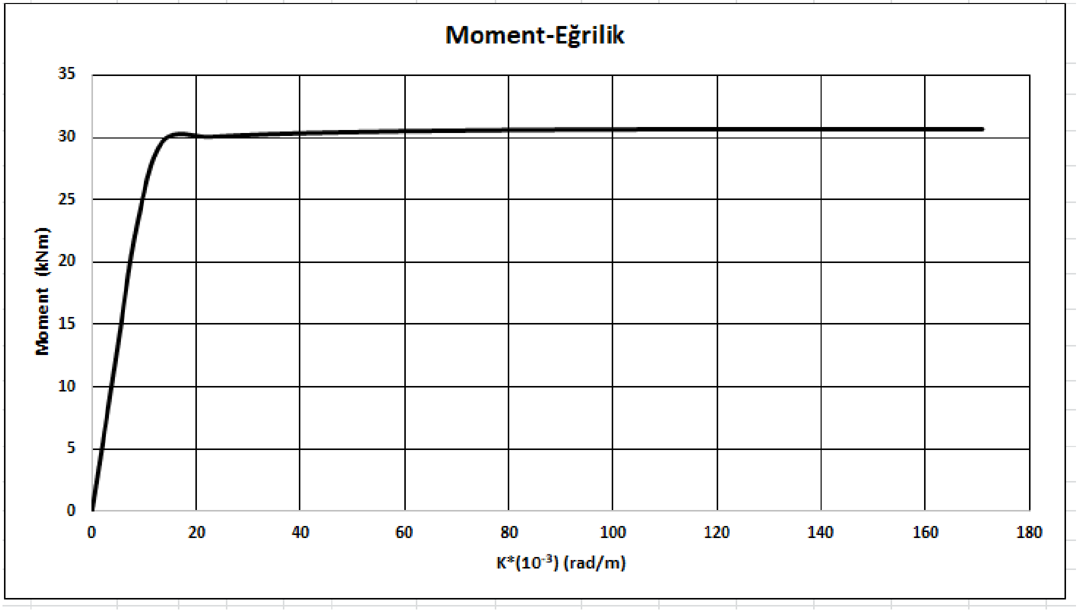
FRP veya çelik donatılı bir betonarme kesitte birim deformasyon, gerilmeler ve iç kuvvetler

$$\phi_M = \frac{\varepsilon_c}{x}$$

$$M = \sum_{i=1}^{n} F_{ci}\,(x - x_i) + (T_f \text{ or } T_s)(x - d) + (C_f \text{ or } C_s)(x - d')$$

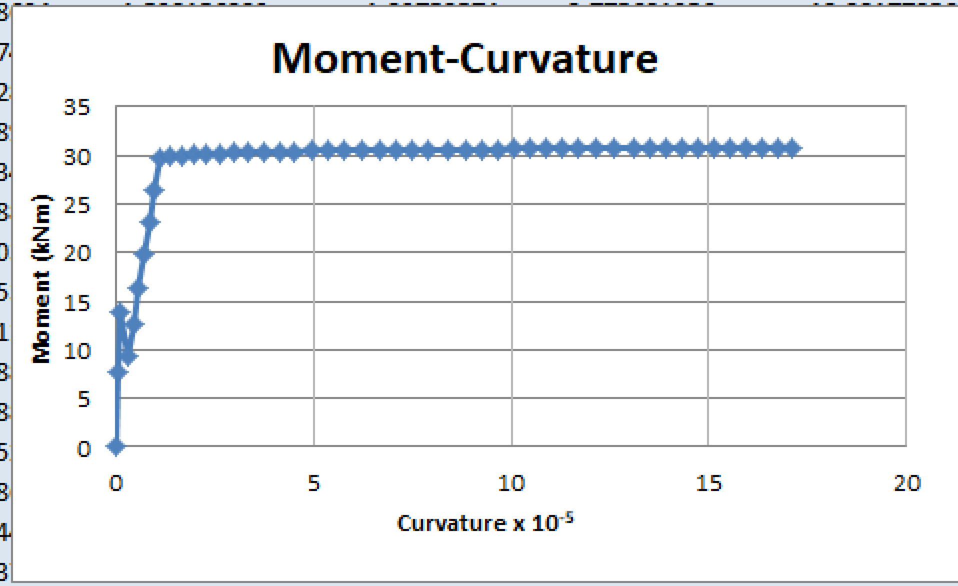
Crackv2_beam.xlsm





MomentEğrilik_R3.xlsm

	A	B	C	D	E	F	G	H
4	BEAM #1							
5	Curvature*1.0e5	Moment (kNm)	NA depth/Depth	Conc. Force (kN)	Comp. Force (kN)	Tens. Force (kN)	Tens. stress (Mpa)	Tolerance(N)
6	0.000468508	0.076844449	0.498035392	-0.015407084	-0.01224723	0.027654313	0.122364219	6.31509E-08
7	0.047181747	7.69031532	0.49948	[illegible]	[illegible]	[illegible]	[illegible]	-7.96108E-06
8	0.098275981	13.97123478	0.477227					-7.82261E-06
9	0.29584031	9.318598124	0.237402					5.66845E-05
10	0.454452211	12.7074416	0.205888					0.000164783
11	0.597149272	16.2616656	0.195763					4.16548E-05
12	0.732790772	19.74923753	0.191368					-1.00533E-05
13	0.863967843	23.15294602	0.189320					-6.79778E-05
14	0.991332074	26.46628067	0.1885					0.000271359
15	1.117423338	29.73916937	0.188141					2.72721E-05
16	1.378471724	29.87186651	0.169438					-1.10692E-05
17	1.664304112	29.96634977	0.154358					-1.57009E-05
18	1.972738287	30.04719596	0.14205					-0.00212394
19	2.297334168	30.12046021	0.132138					0.000333855
20	2.636833328	30.18665653	0.123974					9.17263E-05
21	2.995399298	30.24176862	0.116923					0.000463458
22	3.366620918	30.29103304	0.110961904	-103.9630088	-9.036991493	113	500	-0.000305805
23	3.743907683	30.3363947	0.106012229	-104.0732477	-8.926752962	113	500	-0.000674899
24	4.14397177	30.37346621	0.101408348	-104.2755058	-8.72449405	113	500	0.000102597
25	4.534639536	30.41121114	0.097817404	-104.4398046	-8.560196275	113	500	-0.000844306
26	4.955092014	30.4400418	0.094226303	-104.7244311	-8.275568265	113	500	0.000652959
27	5.381273484	30.46456819	0.091099873	-105.0322039	-7.967795123	113	500	0.000953715
28	5.76554287	30.49864072	0.089075162	-105.1706531	-7.829346805	113	500	0.000130477
29	6.221047576	30.51563636	0.086303793	-105.5968921	-7.403107795	113	500	7.2479E-05
30	6.675166475	30.53069312	0.083927994	-106.0175319	-6.982467441	113	500	0.000655726



	A	B	C	D	E	F	G	H
31	7.080517289	30.55342238	0.082418649	-106.2411495	-6.758850154	113	500	0.000317945
32	7.481350763	30.57465635	0.081121714	-106.4465171	-6.553482919	113	500	-5.63079E-05
33	7.910241905	30.59017518	0.07967308	-106.7652369	-6.234762704	113	500	0.000412637
34	8.394570513	30.59660485	0.077855879	-107.3079243	-5.692075129	113	500	0.000593651
35	8.852935463	30.60446051	0.076460514	-107.7457194	-5.254280731	113	500	-8.91854E-05
36	9.266729319	30.61598292	0.075564237	-108.0034459	-4.996553553	113	500	0.000528467
37	9.675397894	30.6261886	0.074784177	-108.2404678	-4.759532509	113	500	-0.000325314
38	10.08146692	30.63554673	0.074086441	-108.4669871	-4.533013652	113	500	-0.000737997
39	10.48432926	30.64424853	0.073465199	-108.6805497	-4.319449777	113	500	0.000538139
40	10.88393073	30.65228349	0.072911771	-108.8809397	-4.11905984	113	500	0.000505359
41	11.28021415	30.65964075	0.072418838	-109.0679245	-3.932074816	113	500	0.000675728
42	11.67311871	30.66630909	0.071980193	-109.2412605	-3.758740416	113	500	-0.000896858
43	12.14112817	30.66765263	0.071127382	-109.7180186	-3.281982174	113	500	-0.000754655
44	12.61513935	30.66835737	0.070304416	-110.2190222	-2.780977027	113	500	0.000774531
45	13.08631791	30.66897246	0.069556107	-110.7085834	-2.291415662	113	500	0.000891783
46	13.50126237	30.6727013	0.069146621	-110.9709591	-2.029040033	113	500	0.000898771
47	13.91181768	30.67634035	0.068783248	-111.2156043	-1.784396572	113	500	-0.000891523
48	14.32019439	30.67974883	0.068451119	-111.4514446	-1.548554666	113	500	0.000775202
49	14.72635663	30.68292236	0.068147655	-111.6783416	-1.321659218	113	500	-0.000816071
50	15.1302674	30.68585638	0.067870578	-111.8961414	-1.103859708	113	500	-0.001091366
51	15.53188716	30.68854624	0.067617883	-112.1046842	-0.895315864	113	500	-6.34142E-05
52	15.93146278	30.69102414	0.067386572	-112.3049688	-0.695030369	113	500	0.000810735
53	16.32997695	30.69341658	0.067170946	-112.500967	-0.499033105	113	500	-0.00012904
54	16.72750559	30.69573358	0.066969539	-112.6929819	-0.307017423	113	500	0.000698891
55	17.12403693	30.69797401	0.066781371	-112.8809695	-0.119030814	113	500	-0.000344658

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Prediction of load–deflection behavior of multi-span FRP and steel reinforced concrete beams

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ABSTRACT

This paper presents a numerical procedure to determine the deflection of concrete members reinforced with fiber reinforced polymer (FRP) or steel bars. This procedure is implemented into the stiffness matrix to allow for general use in the structural analysis. It considers effective flexibilities of members in the cracked state using either the curvature distribution along the member or available effective stiffness models under any loading or support condition. In general, structural concrete members can be considered to have three cracked regions (two at the ends and one at midspan) and two uncracked regions along their length. In this numerical procedure, the contributions of these regions to the member stiffness matrix are computed using a numerical integration technique. Using this procedure, a software program is developed which allows for the load–deflection behavior of a member reinforced with either FRP or steel bars and subjected to any loading or support condition to be rapidly determined. This calculation procedure is evaluated using available experimental data on the load–deflection behavior of simple and two-span beams reinforced with FRP and steel bars. Through comparison of the results, it is observed that the load–deflection behaviors calculated using the proposed approach utilizing the member moment–curvature response are consistent with the experimental data. This approach can provide a useful tool for the general calculation of deflection regardless of reinforcement type and can be used throughout the entire range of member behavior up to flexural failure.