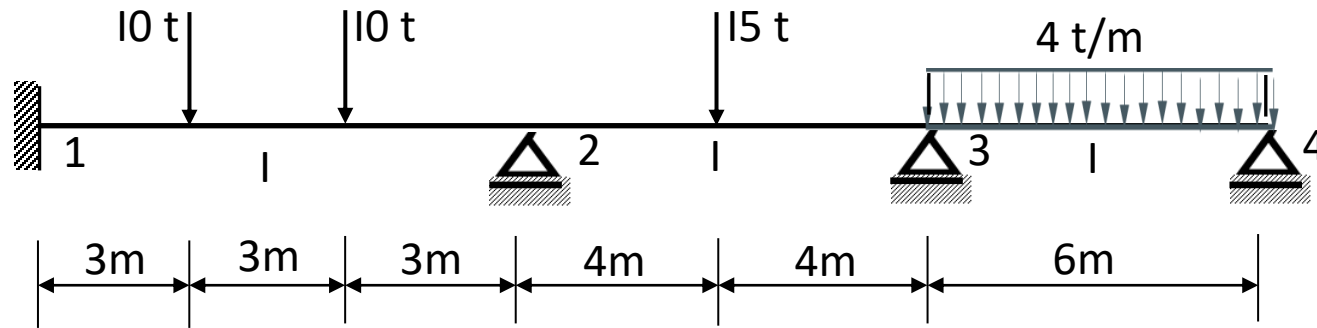


YAPI STATİĞİ 2-HİPERSTATİK SİSTEMLER DEPLASMAN METODLARI(2-4)

Prof. Dr. Cengiz DüNDAR

ÖRNEK 1

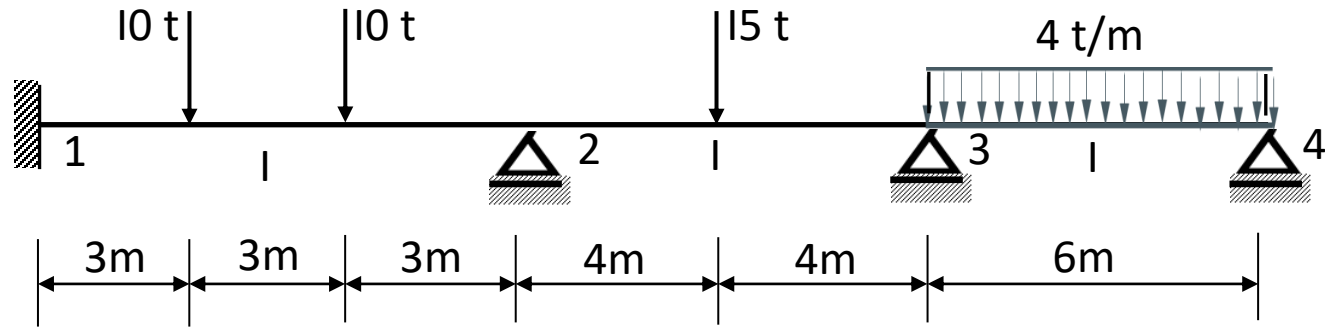


1. Ankastrelik momentleri

$$M_{12} = -M_{21} = \frac{2}{9} PL = \frac{2}{9} * 10 * 9 = 20 \text{ tm}$$

$$M_{23} = -M_{32} = \frac{PL}{8} = \frac{15 * 8}{8} = 15 \text{ tm}$$

$$M_{34} = \frac{qL^2}{8} = \frac{4 * 6^2}{8} = 18 \text{ tm}$$

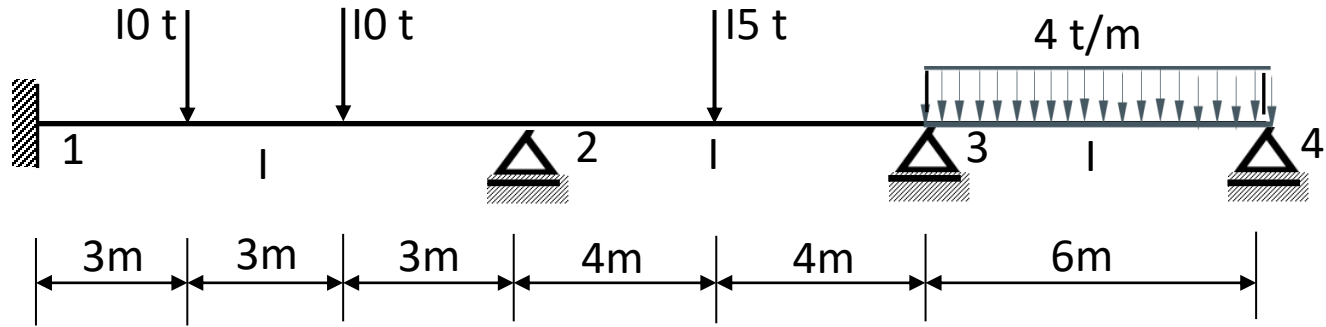


2. Dağıtma sayıları

$$r_{21} = \frac{m_{2\theta_2}^{21}}{m_{2\theta_2}^{21} + m_{2\theta_2}^{23}} = \frac{\frac{4EI}{9}}{\frac{4EI}{9} + \frac{4EI}{8}} = \frac{0.111}{0.111 + 0.125} = 0.47$$

$$r_{23} = \frac{m_{2\theta_2}^{23}}{m_{2\theta_2}^{21} + m_{2\theta_2}^{23}} = \frac{\frac{4EI}{8}}{\frac{4EI}{9} + \frac{4EI}{8}} = \frac{0.125}{0.111 + 0.125} = 0.53$$

$$\sum r = 1 \text{ olmalı} \rightarrow 0.47 + 0.53 = 1 \quad \checkmark$$



2. Dağıtma sayıları

$$r_{32} = \frac{m_{3\theta_3}^{32}}{m_{3\theta_3}^{32} + m_{3\theta_3}^{34}} = \frac{\frac{4EI}{8}}{\frac{4EI}{8} + \frac{3EI}{6}} = \frac{0.125}{0.125 + 0.125} = 0.50$$

$$r_{34} = \frac{m_{3\theta_3}^{34}}{m_{3\theta_3}^{32} + m_{3\theta_3}^{34}} = \frac{\frac{3EI}{6}}{\frac{4EI}{8} + \frac{3EI}{6}} = \frac{0.125}{0.125 + 0.125} = 0.50$$

$$\sum r = 1 \text{ olmalı} \rightarrow 0.50 + 0.50 = 1 \quad \checkmark$$

3. Dengeleme

	$\frac{1}{2}$	0.47	0.53	$\frac{1}{2}$	0.50	0.50	0	
1		2			3			4
20.0		-20.0	15.0		-15.0	18.0		
1.18	←	2.35	2.65	→	1.33			
			-1.09	←	-2.17	-2.17		
0.26	←	0.51	0.58	→	0.29			
			-0.08	←	-0.15	-0.15		
0.02	←	0.04	0.05	→	0.03			
			-0.01	←	-0.02	-0.02		
<hr/>		<hr/>	<hr/>		<hr/>	<hr/>		
21.46		-17.10	17.11		-15.68	15.66		

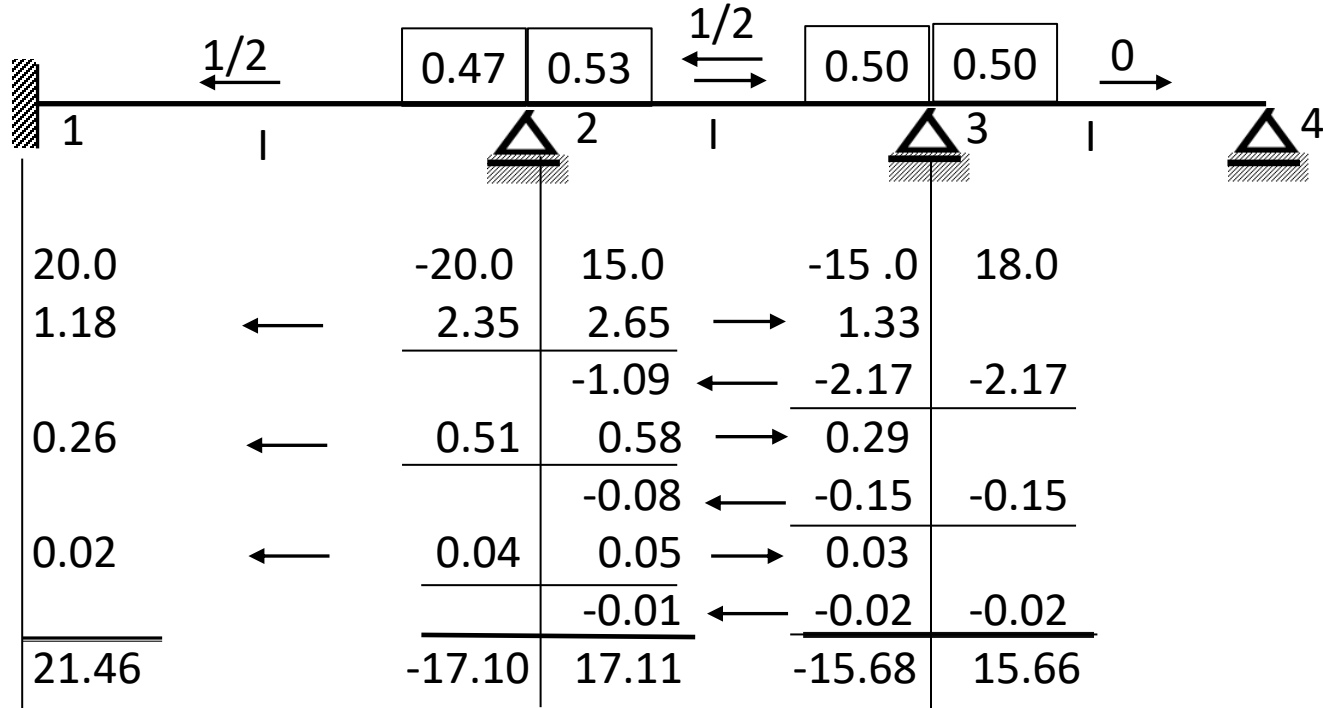
Fark momentin büyük olduğu düğümden başlanır

2 düğüm noktasının kilidini çözelim $M_i + M_k = 0 \rightarrow -M_k = M_i \rightarrow M_2 = -20 + 15 = -5 \text{ tm}$
 $-(-5.0) * 0.47 = 2.35 \text{ tm} \quad -(-5.0) * 0.53 = 2.65 \text{ tm}$

3 düğüm noktasının kilidini çözüp 2 düğüm noktasını kilitleyelim

$M_i + M_k = 0 \rightarrow -M_k = M_i \rightarrow M_3 = -15 + 18 + 1.33 = 4.33 \text{ tm}$
 $-(4.33 * 0.50) = -2.17 \text{ tm} \quad -(4.33) * 0.50 = -2.17 \text{ tm}$

3. Dengeleme



3 düğüm noktasının kilitleyip 2 düğüm noktasının kilidini çözelim

$$M_2 = -1.09 \text{ tm}$$

$$-(-1.09 * 0.47) = 0.51 \text{ tm} \quad -(-1.09) * 0.53 = 0.58 \text{ tm}$$

3. Dengeleme

	$\frac{1}{2}$ ←	0.47	0.53	← $\frac{1}{2}$	0.50	0.50	→ 0
20.0		-20.0	15.0		-15.0	18.0	
1.18	←	2.35	2.65	→	1.33		
			-1.09	←	-2.17	-2.17	
0.26	←	0.51	0.58	→	0.29		
			-0.08	←	-0.15	-0.15	
0.02	←	0.04	0.05	→	0.03		
			-0.01	←	-0.02	-0.02	
<hr/>		<hr/>	<hr/>		<hr/>	<hr/>	
21.46		-17.10	17.11		-15.68	15.66	

3 düğüm noktasının kilidini çözüp 2 düğüm noktasını kilitleyelim

$$M_3 = 0.29tm$$

$$-(0.29 * 0.50) = -0.15 tm \quad - (0.29) * 0.50 = -0.15 tm$$

3. Dengeleme

		1/2			1/2			0	
		0.47	0.53		0.50	0.50			
		△			△			△	
		2			3			4	
20.0		-20.0	15.0		-15.0	18.0			
1.18	←	2.35	2.65	→	1.33				
			-1.09	←	-2.17	-2.17			
0.26	←	0.51	0.58	→	0.29				
			-0.08	←	-0.15	-0.15			
0.02	←	0.04	0.05	→	0.03				
			-0.01	←	-0.02	-0.02			
21.46		-17.10	17.11		-15.68	15.66			

4. Süperpozisyon yaparak çubuk uç momentlerinin hesabı

$$M_{12} = 20.0 + 1.18 + 0.26 + 0.02 = 21.46 \text{ tm}$$

$$M_{21} = -20.0 + 2.35 + 0.51 + 0.04 = -17.10 \text{ tm}$$

$$M_{23} = 15 + 2.65 - 1.09 + 0.58 - 0.08 + 0.05 - 0.01 = 17.11 \text{ tm}$$

$$M_{32} = -15.0 + 1.33 - 2.17 + 0.29 - 0.15 + 0.03 - 0.02 = -15.68 \text{ tm}$$

$$M_{34} = 18.0 - 2.17 - 0.15 - 0.02 = 15.66 \text{ tm}$$

3. Dengeleme

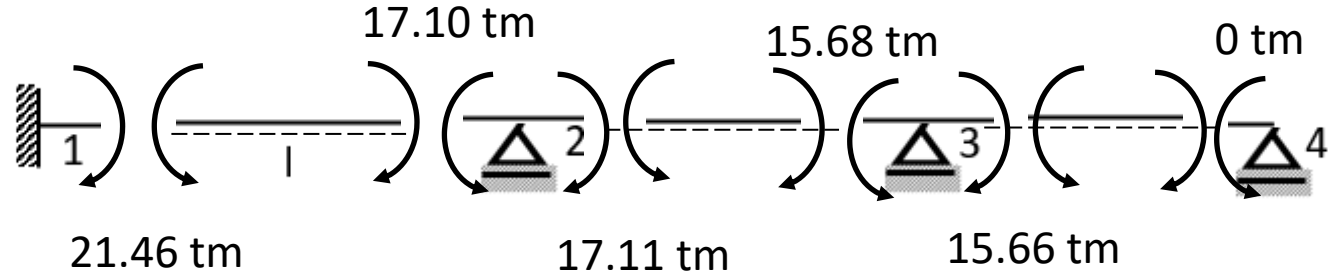
	$\frac{1}{2}$	0.47	0.53	$\frac{1}{2}$	0.50	0.50	0	
1		2			3			4
20.0		-20.0	15.0		-15.0	18.0		
1.18	←	2.35	2.65	→	1.33			
			-1.09	←	-2.17	-2.17		
0.26	←	0.51	0.58	→	0.29			
			-0.08	←	-0.15	-0.15		
0.02	←	0.04	0.05	→	0.03			
			-0.01	←	-0.02	-0.02		
<hr/>		<hr/>	<hr/>		<hr/>	<hr/>		
21.46		-17.10	17.11		-15.68	15.66		

5. 2 ve 3 nolu düğüm noktalarının dönmeleri hesabı

$$\theta_2 = \frac{2.35 + 0.51 + 0.04}{m_{2\theta_2}^{21}} = \frac{2.5}{\frac{4EI}{9}} = \frac{6.52}{EI} rd \quad \theta_2 = \frac{2.65 + 0.58 + 0.05}{m_{2\theta_2}^{23}} = \frac{3.28}{\frac{4EI}{8}} = \frac{6.52}{EI} rd \quad \checkmark$$

$$\theta_3 = \frac{-2.17 - 0.15 - 0.02}{m_{3\theta_3}^{32}} = \frac{-2.34}{\frac{4EI}{8}} = \frac{-4.68}{EI} rd \quad \theta_3 = \frac{-2.17 - 0.15 - 0.02}{m_{3\theta_3}^{34}} = \frac{-2.34}{\frac{3EI}{6}} = \frac{-4.68}{EI} rd$$

4. Dönüştürme



7. Kesme kuvvetleri hesabı

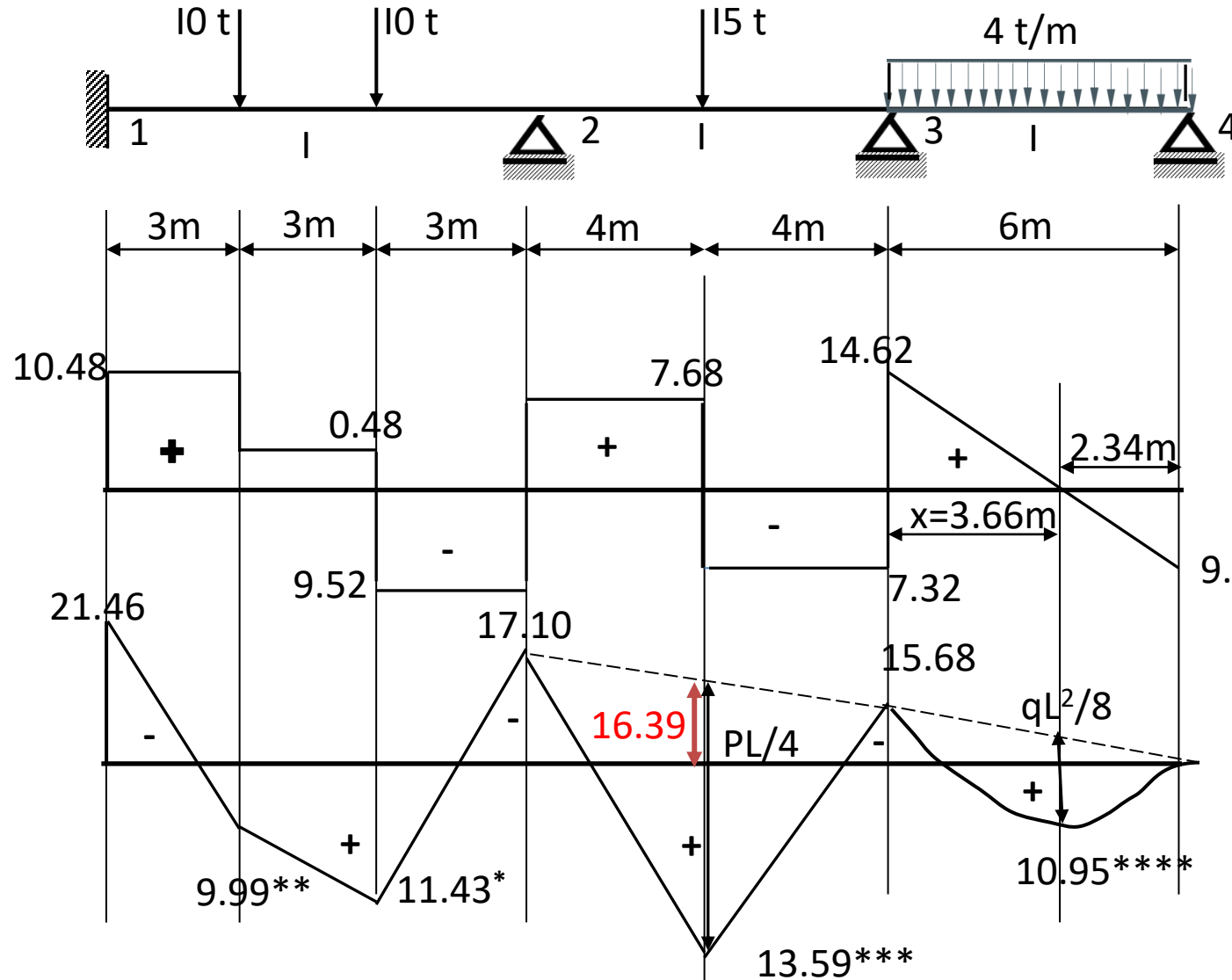
$$T_{1sağ} = T_{0ij} + \frac{M_{ij} + M_{ji}}{L} = 10 + \frac{21.46 - 17.10}{9} = 10.48 t \quad T_{2sol} = -10 + \frac{21.46 - 17.10}{9} = -9.52 t$$

$$T_{2sağ} = 7.5 + \frac{17.11 - 15.68}{8} = 7.5 + 0.18 = 7.68 t$$

$$T_{3sol} = -7.5 + \frac{17.11 - 15.68}{8} = -7.5 + 0.18 = -7.32 t \quad T_{3sağ} = 12 + \frac{15.66}{6} = 14.62 t$$

$$T_{4sol} = -12 + \frac{15.66}{6} = -9.38 t$$

5. Kesme kuvveti ve moment diyagramları



$$*** \frac{17.10 - 15.68}{2} + 15.68 = 16.39$$

$$*** M_{max} = \frac{15 * 8}{4} - 16.39 = 13.61 tm$$

Kesme Kuvveti Diyagramı

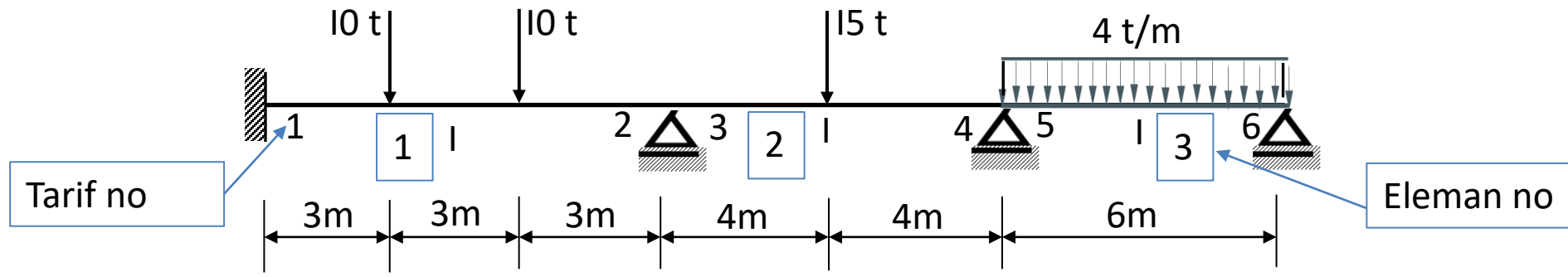
$$\frac{14.62}{x} = \frac{9.38}{6 - x} \rightarrow x = 3.66 m$$

Moment Diyagramı

$$**** M_{max} = \frac{1}{2} * 9.38 * 2.34 = 10.95 tm$$

$$* M_{max} = -21.46 + 10.48 * 3 + 0.48 * 3 = 11.42 tm$$

$$** M = -21.46 + 10.48 * 3 = 9.98 tm$$



*** C R O S S - Y O N T E M I ***1994

Prof.Dr. Cengiz DUNDAR

Ars.Gor. Bulent SAHIN

3 ACIKLIKLI SUREKLI KIRIS

V E R I D E G E R L E R I :

ELEMAN	BOY	ATALET MOM	DAGITMA KATS
1	9.00	1.000	0.000 0.471
2	8.00	1.000	0.529 0.500
3	6.00	1.000	0.500 1.000

YUKLEME NO= 1

ITERASYON SAYISI=10

ELEMAN	TARIF NO	ANKAS.UC MOM.	BAS.KI.KESME K.
1	1	-20.00	-10.00
	2	20.00	10.00
2	3	-15.00	-7.50
	4	15.00	7.50
3	5	-12.00	-12.00
	6	12.00	12.00

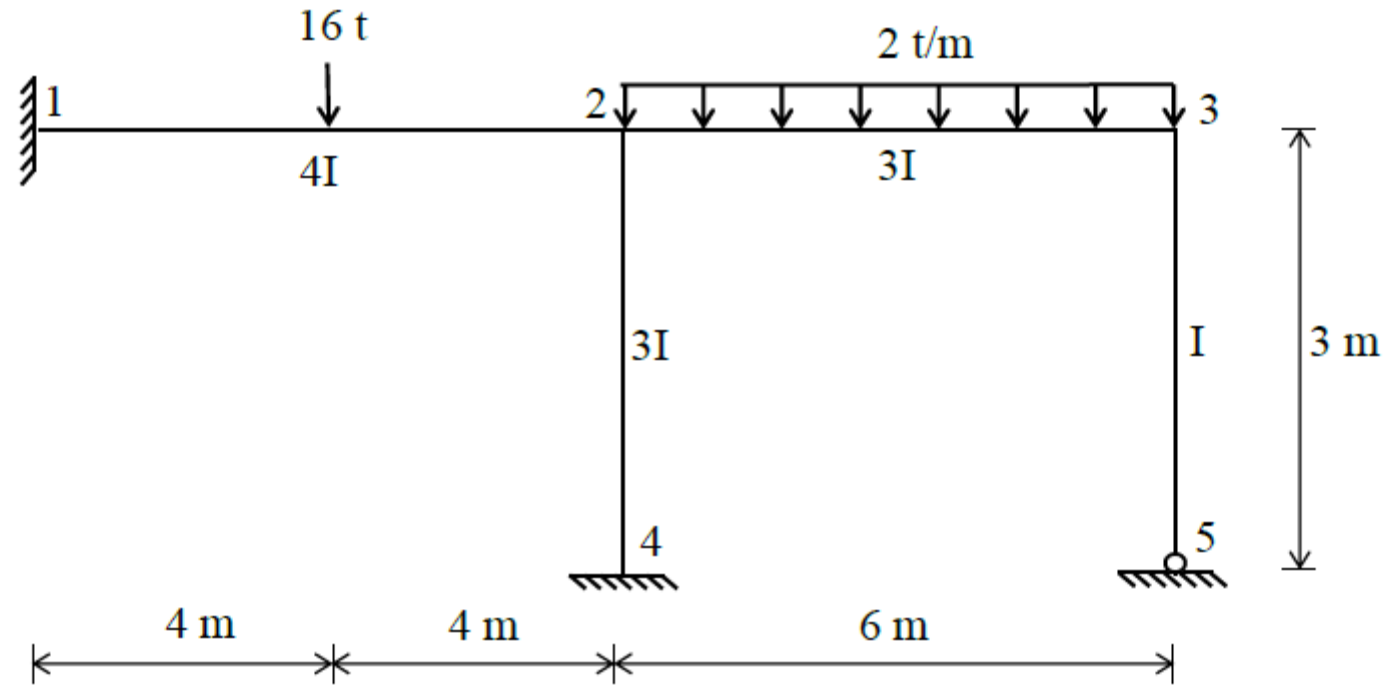
HESAPLANAN DEGERLER:

*** E L E M A N U C K U V V E T L E R I ***

ELEMAN	TARIF NO	MOMENT	KESME KUVVETI
1	1	-21.449	-10.483
	2	17.102	9.517
2	3	-17.102	-7.677
	4	15.685	7.323
3	5	-15.685	-14.614
	6	0.000	9.386

Xmax(i ucundan)= 3.000
1INCI ELEMAN ACIKLIK MOMENTI= 10.000
Xmax(i ucundan)= 4.000
2INCI ELEMAN ACIKLIK MOMENTI= 13.606
Xmax(i ucundan)= 3.654
3INCI ELEMAN ACIKLIK MOMENTI= 11.012

ÖRNEK 2



1. Ankastrelik Momentler

$$-M_{21} = M_{12} = \frac{Pl}{8} = \frac{16 \times 8}{8} = 16 \text{ tm}$$

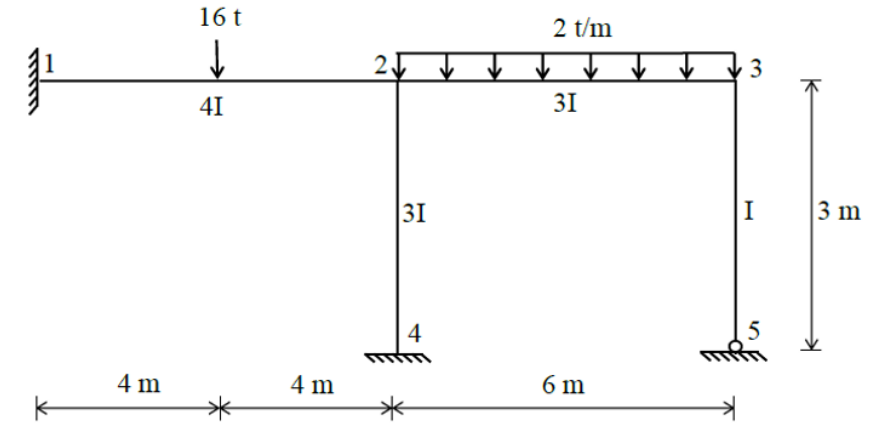
$$M_{23} = -M_{32} = \frac{ql^2}{12} = \frac{2 \times 6^2}{12} = 6 \text{ tm}$$

2. Dağıtma Sayıları

$$r_{21} = \frac{m_{2\theta_2}^{21}}{m_{2\theta_2}^{21} + m_{2\theta_2}^{23} + m_{2\theta_2}^{24}} = \frac{\frac{4E(4I)}{8}}{\frac{4E(4I)}{8} + \frac{4E(3I)}{6} + \frac{4E(3I)}{3}} = 0.25$$

$$r_{23} = \frac{\frac{4E(3I)}{6}}{\frac{4E(4I)}{8} + \frac{4E(3I)}{6} + \frac{4E(3I)}{3}} = 0.25$$

$$r_{24} = \frac{\frac{4E(3I)}{3}}{\frac{4E(4I)}{8} + \frac{4E(3I)}{6} + \frac{4E(3I)}{3}} = 0.50$$



$$\sum r = 1$$

$$r_{32} = \frac{\frac{4E(3I)}{6}}{\frac{4E(3I)}{6} + \frac{3E(I)}{3}} = 0.667$$

$$r_{35} = \frac{\frac{3E(I)}{3}}{\frac{4E(3I)}{6} + \frac{3E(I)}{3}} = 0.333$$

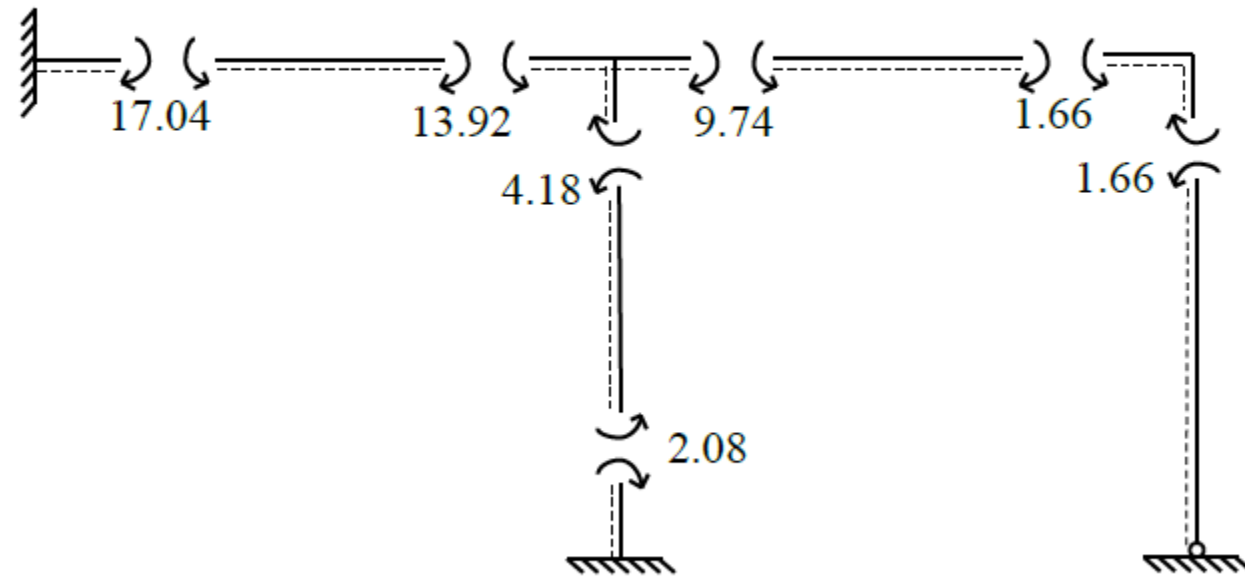
$$\sum r = 1$$

3. Cross Dengelemeleri

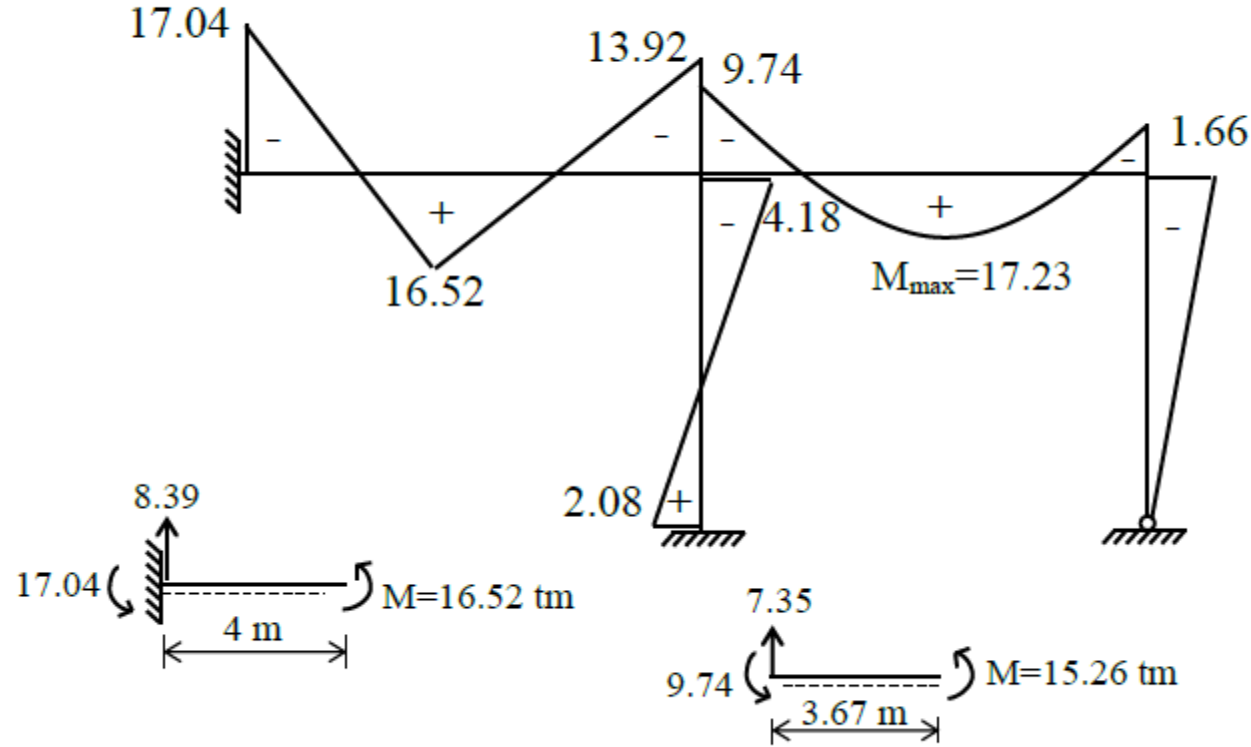
Fark momentin büyük olduğu düğümden başlanır

	(21)	(24)	(23)	(32)	(35)	
1/2 ←	0.25		0.25		0.667	1/2 →
		0.50			0.333	
16.00	-16.00	0	0	6.00	-6.00	
1.25 ←	2.50	5.00	2.50	1.25	1.25	-6+1.25=-4.75
			1.59	3.17	1.59	$-(-4.75) \times 0.667 = 3.17$
-0.20 ←	-0.40	-0.79	-0.40	-0.20	-0.20	$-(-4.75) \times 0.333 = 1.59$
			0.07	0.13	0.07	
-0.01 ←	-0.02	-0.03	-0.02	-0.01	-0.01	
17.04	-13.92	4.18	9.74		1.66	
		2.50				
		-0.40				
		-0.02				
		2.08				

4. Dönüştürme



5. Moment Diyagramı



6. Uç Kesme Kuvvetlerin Hesabı

$$T_{12} = 8 + \frac{17.04 - 13.92}{8} = 8.39 \text{ t}$$

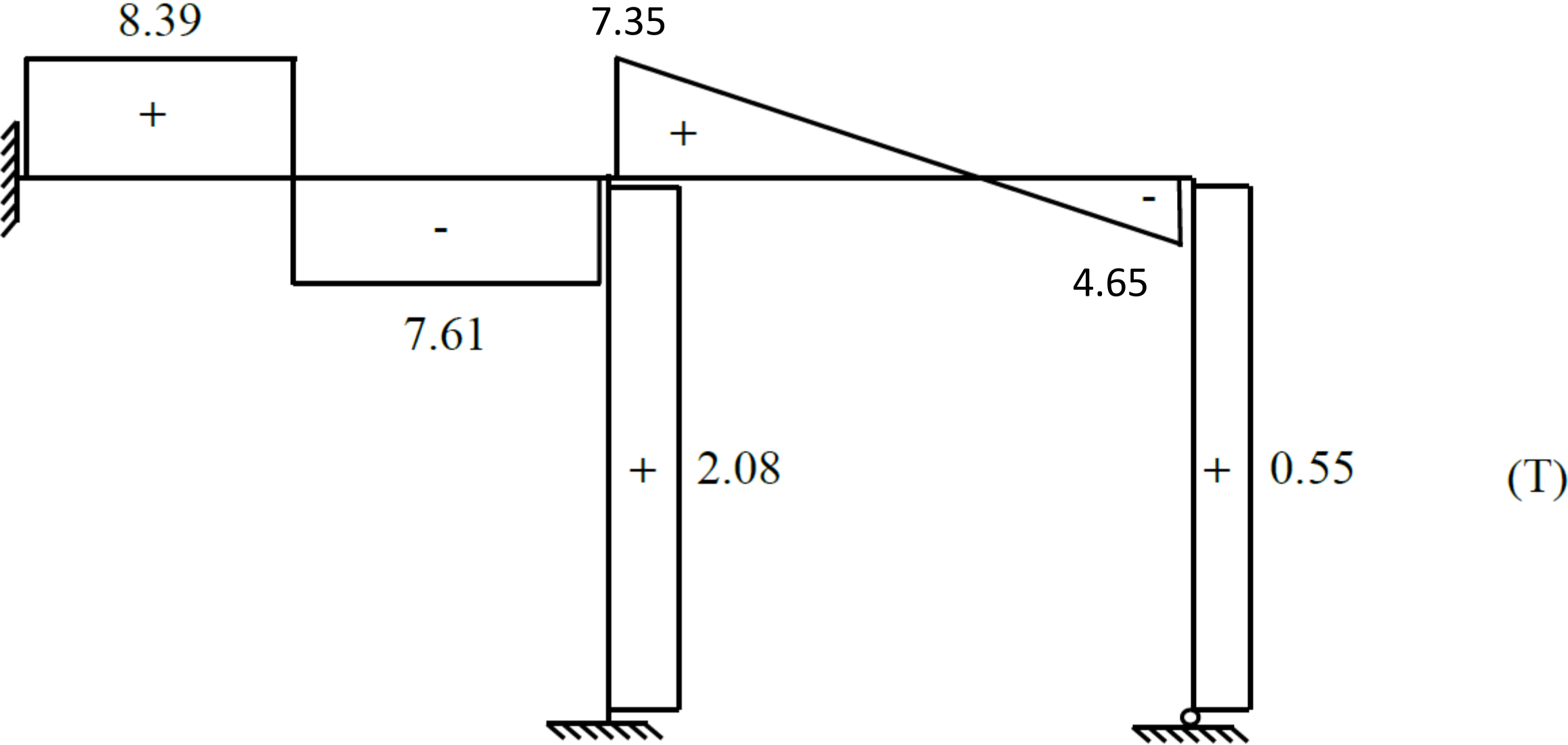
$$T_{21} = -8 + 0.39 = -7.61 \text{ t}$$

$$T_{23} = 6 + \frac{9.74 - 1.66}{6} = 7.35 \text{ t}$$

$$T_{32} = -6 + 1.35 = -4.65 \text{ t}$$

$$M = -9.74 + 7.35 \times 3.67 = 17.23 \text{ tm}$$

6. Kesme kuvveti diyagramı



21 çubuğu

$$\Delta\theta_2 = \frac{2.50}{m_{2\theta_2}^{21}} = \frac{1.25}{EI} \quad , \quad \frac{-0.40}{2EI} = \frac{-0.2}{EI} \quad , \quad \frac{-0.02}{2EI} = \frac{-0.01}{EI}$$

\searrow
 $\frac{4EI}{L} = \frac{4E(4I)}{8} = 2EI$

$$\theta_2 = \frac{1.25}{EI} - \frac{0.2}{EI} - \frac{0.01}{EI} = \frac{1.04}{EI}$$

23 çubuğu

$$\theta_2 = \frac{2.50 - 0.4 - 0.02}{2EI} = \frac{1.04}{EI}$$

	1/2 ←	(21) 0.25		(23) 0.25	↔ 1/2	(32) 0.667		(35) 0.333
			0.50					
16.00		-16.00	0	0	6.00	-6.00		
1.25 ←		2.50	5.00	2.50	→	1.25		
-0.20 ←		-0.40	-0.79	-0.40	←	3.17	1.59	
-0.01 ←		-0.02	-0.03	-0.02	←	0.13	0.07	
17.04		-13.92	4.18	9.74				1.66
			2.50					
			-0.40					
			-0.02					
			2.08					

21 çubuğu

$$\Delta\theta_2 = \frac{2.50}{m_{2\theta_2}^2} = \frac{1.25}{EI}, \quad \frac{-0.40}{2EI} = \frac{-0.2}{EI}, \quad \frac{-0.02}{2EI} = \frac{-0.01}{EI}$$

↙

$$\frac{4EI}{L} = \frac{4E(4I)}{8} = 2EI$$

$$\theta_2 = \frac{1.25}{EI} - \frac{0.2}{EI} - \frac{0.01}{EI} = \frac{1.04}{EI}$$

23 çubuğu

$$\theta_2 = \frac{2.50 - 0.4 - 0.02}{2EI} = \frac{1.04}{EI}$$

	$1/2 \leftarrow$	<u>(21)</u> 0.25	<u>(24)</u> 0.50	<u>(23)</u> 0.25	$\rightarrow 1/2$	<u>(32)</u> 0.667	<u>(35)</u> 0.333
16.00		-16.00	0	0		-6.00	
1.25	\leftarrow	<u>2.50</u>	5.00	<u>2.50</u>	\rightarrow	1.25	
-0.20	\leftarrow	<u>-0.40</u>	-0.79	<u>-0.40</u>	\rightarrow	<u>3.17</u>	1.59
-0.01	\leftarrow	<u>-0.02</u>	-0.03	<u>-0.02</u>	\rightarrow	<u>0.13</u>	0.07
17.04		<u>-13.92</u>	4.18	9.74		<u>-0.01</u>	1.66
			2.50				
			-0.40				
			<u>-0.02</u>				
			2.08				

24 çubuğu

$$\theta_2 = \frac{5.0 - 0.79 - 0.03}{4EI} = \frac{1.045}{EI}$$

32 çubuğu

$$\theta_3 = \frac{3.17 + 0.13}{2EI} = \frac{1.65}{EI}$$

35 çubuğu

$$\theta_3 = \frac{1.59 + 0.07}{EI} = \frac{1.66}{EI}$$

UYGULAMA 1

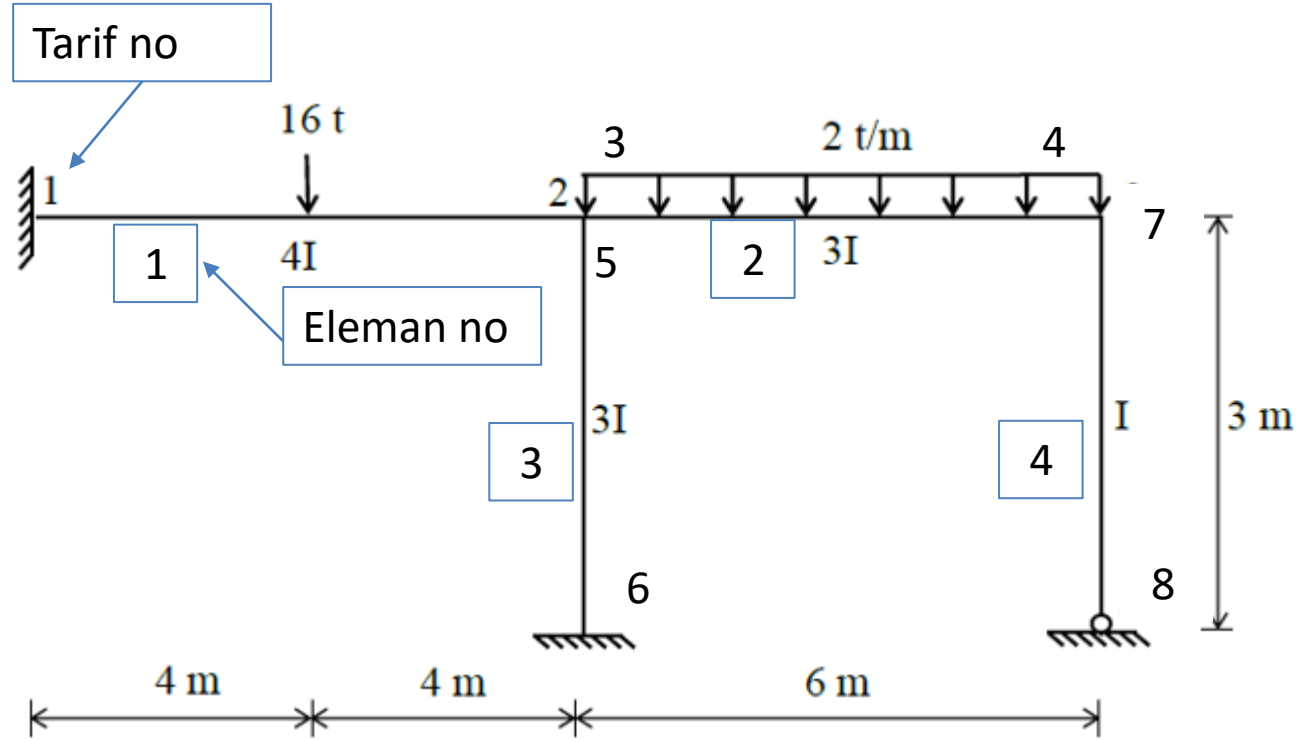
V E R I D E G E R L E R I :

ELEMAN	BOY	ATALET MOM	DAGITMA KATS
1	8.00	4.000	0.000 0.250
2	6.00	3.000	0.250 0.667
3	3.00	3.000	0.500 0.000
4	3.00	1.000	0.333 1.000

YUKLEME NO= 1

ITERASYON SAYISI=10

ELEMAN	TARIF NO	ANKAS.UC MOM.	BAS.KI.I
1	1	-16.00	-8.00
	2	16.00	8.00
2	3	-6.00	-6.00
	4	6.00	6.00
3	5	0.00	0.00
	6	0.00	0.00
4	7	0.00	0.00
	8	0.00	0.00



HESAPLANAN DEGERLER:

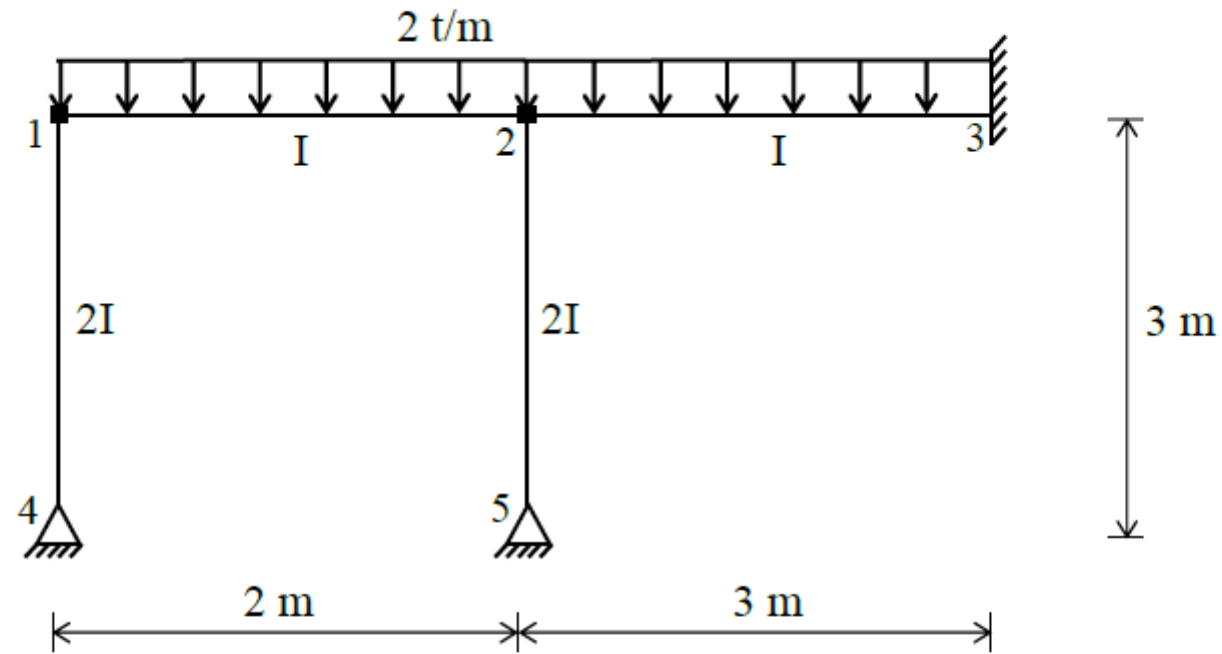
*** E L E M A N U C K U V V E T L E R I ***

ELEMAN	TARIF NO	MOMENT	KESME KUVVETI
1	1	-17.043	-8.391
	2	13.913	7.609
2	3	-9.739	-7.348
	4	1.652	4.652
3	5	-4.174	-2.087
	6	-2.087	-2.087
4	7	-1.652	-0.551
	8	0.000	-0.551

Xmax(i ucundan)= 4.000

1INCI ELEMAN ACIKLIK MOMENTI= 16.522

ÖRNEK- 3



Ankastrelük Momentler

$$M_{12} = -M_{21} = \frac{qL^2}{12} = \frac{2 \times 4}{12} = 0.667 \text{ tm}$$

$$M_{23} = -M_{32} = \frac{qL^2}{12} = \frac{2 \times 9}{12} = 1.5 \text{ tm}$$

1 düğümü

$$r_{12} = \frac{m_{1\theta_1}^{12}}{m_{1\theta_1}^{12} + m_{1\theta_1}^{14}} = \frac{\frac{4EI}{L}}{\frac{4EI}{L} + \frac{3EI}{L}} = \frac{\frac{4EI}{2}}{\frac{4EI}{2} + \frac{3E(2I)}{3}} = 0.5$$

$$r_{14} = \frac{m_{1\theta_1}^{14}}{m_{1\theta_1}^{14} + m_{1\theta_1}^{12}} = \frac{2EI}{4EI} = 0.5$$

$$r_{12} + r_{14} = 0.5 + 0.5 = 1$$

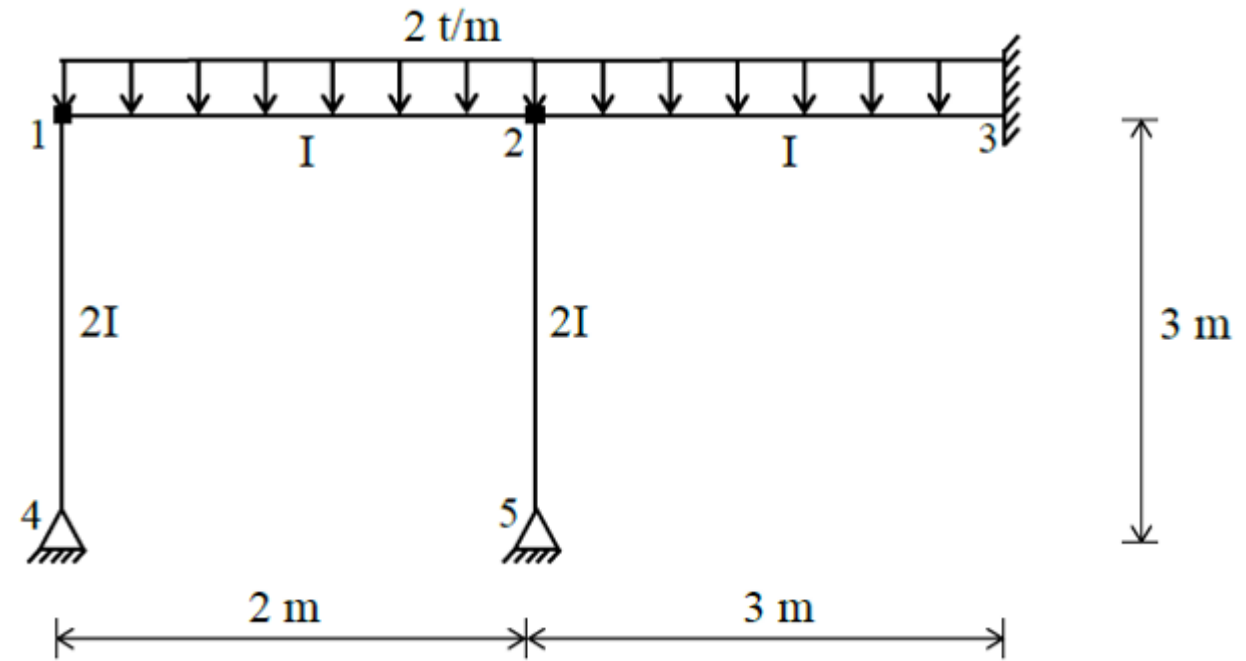
2 düğümü

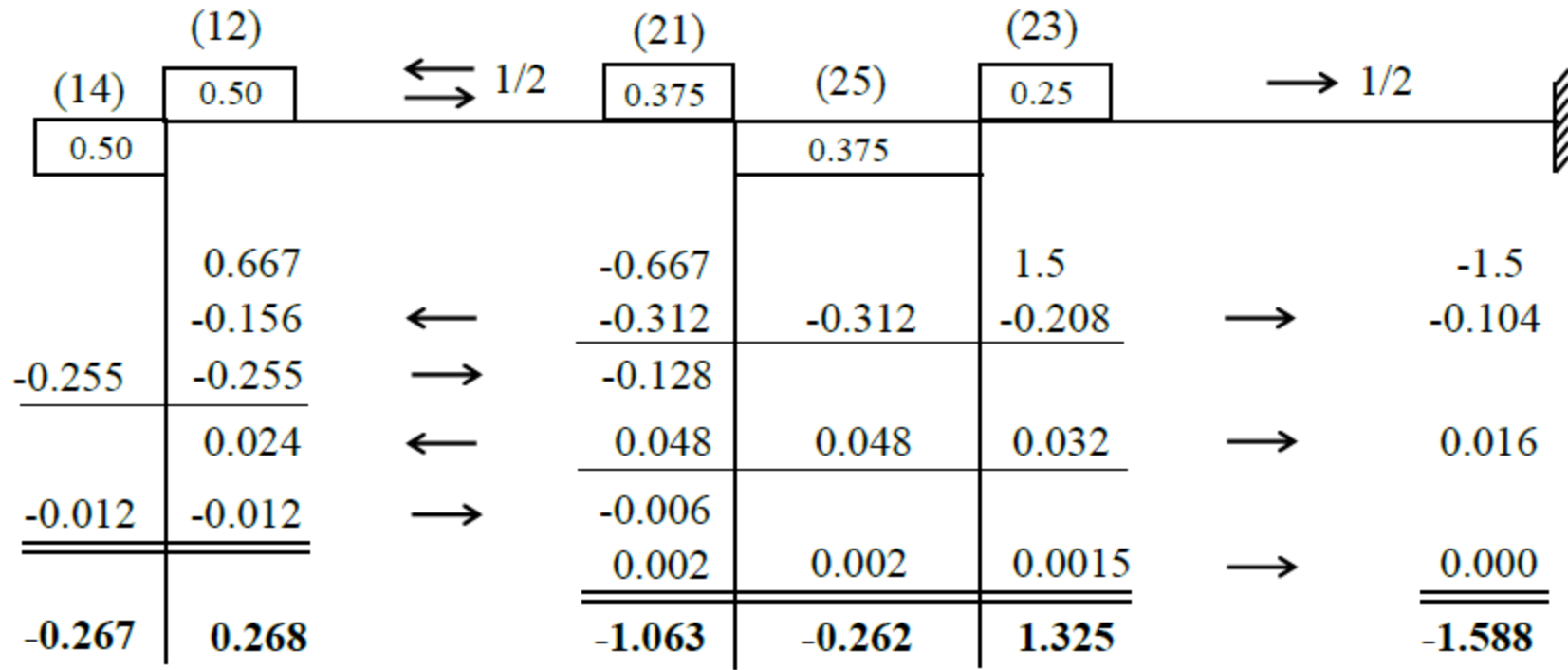
$$r_{21} = \frac{m_{2\theta_2}^{21}}{m_{2\theta_2}^{21} + m_{2\theta_2}^{23} + m_{2\theta_2}^{25}} = \frac{\frac{4EI}{L}}{\frac{4EI}{L} + \frac{4EI}{L} + \frac{3EI}{L}} = \frac{\frac{4EI}{2}}{\frac{4EI}{2} + \frac{4EI}{3} + \frac{3E(2I)}{3}} = \frac{2EI}{5.333EI} = 0.375$$

$$r_{23} = \frac{1.333EI}{5.333EI} = 0.25$$

$$r_{25} = \frac{2EI}{5.333EI} = 0.375$$

$$r_{21} + r_{23} + r_{25} = 1.0$$





Fark momentin büyük olduğu düğümden başlanır

$$2 \rightarrow -0.667 + 1.5 = 0.833$$

$$-0.833 \times 0.375 = -0.312$$

$$1 \rightarrow 0.667 - 0.156 = 0.511$$

$$-0.511 \times 0.5 = -0.255$$

$$\theta_1^{14} = \frac{-0.267}{\frac{3EI}{L}} = \frac{-0.267}{2EI} = -0.133/EI$$

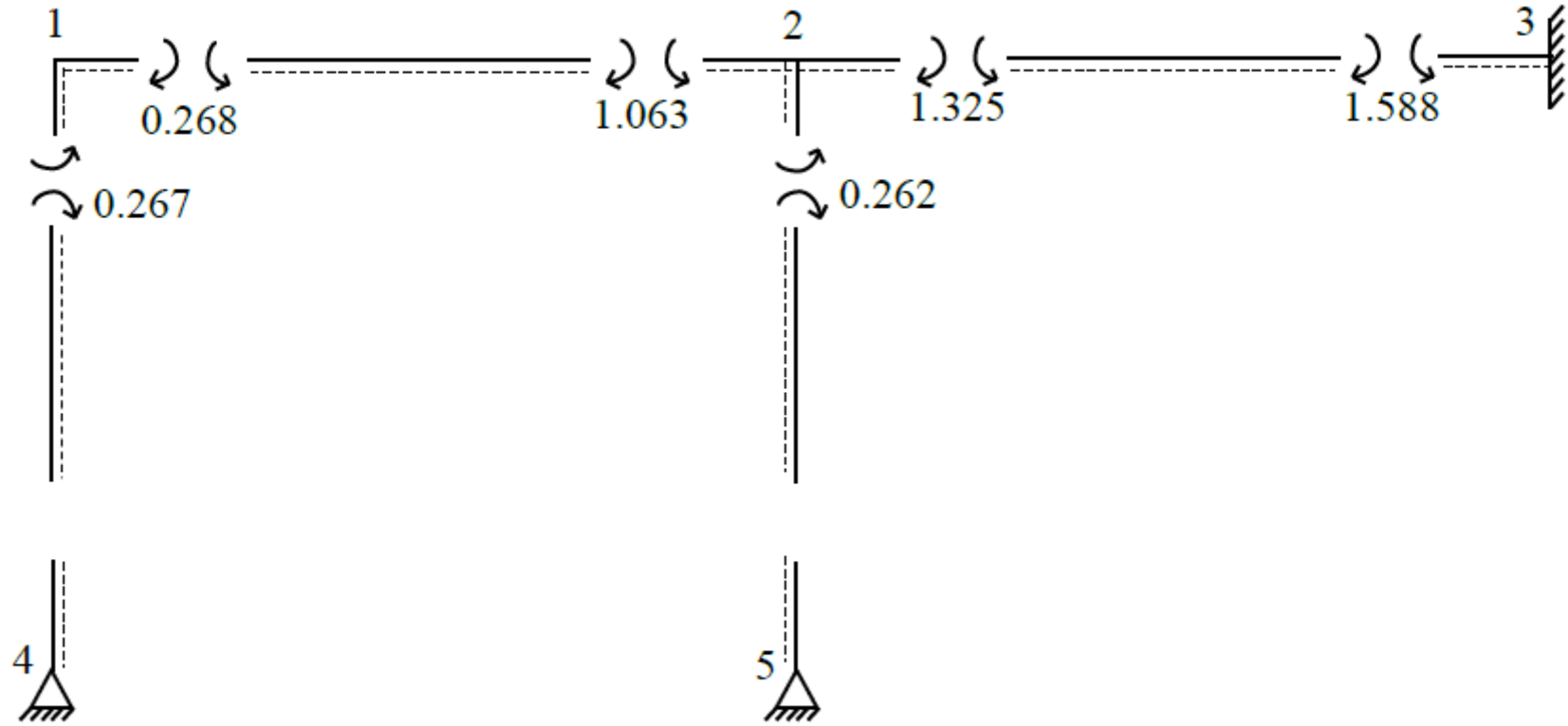
$$\theta_1^{12} = -0.133/EI$$

$$\theta_2^{21} = \frac{-0.312+0.048+0.002}{\frac{4EI}{L}} = \frac{-0.267}{2EI} = -0.131/EI$$

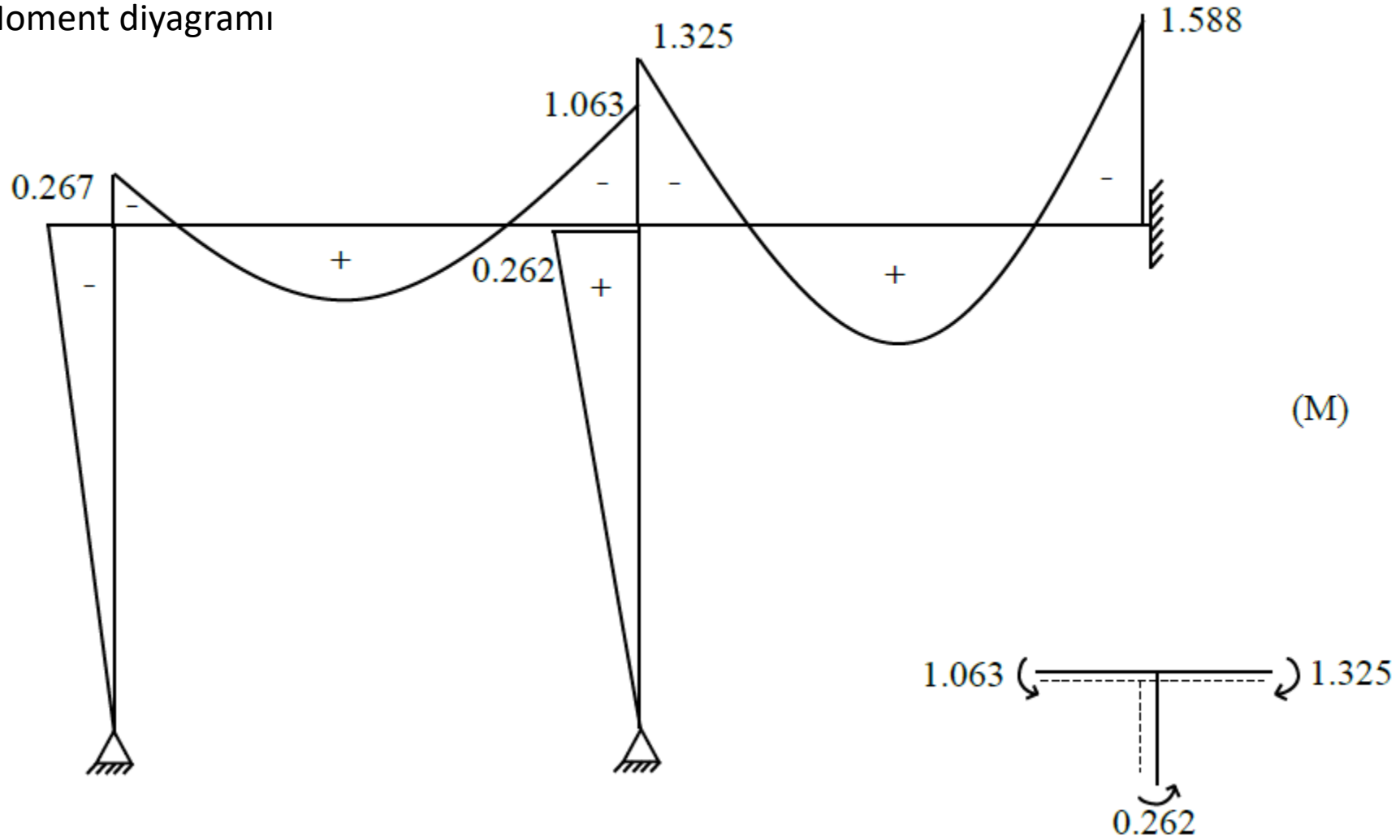
$$\theta_2^{25} = \frac{-0.312+0.048+0.002}{\frac{3EI}{L}} = \frac{-0.267}{2EI} = -0.131/EI$$

$$\theta_2^{23} = \frac{-0.208+0.032+0.0015}{\frac{4EI}{L}} = \frac{-0.1745}{1.333EI} = -0.131/EI$$

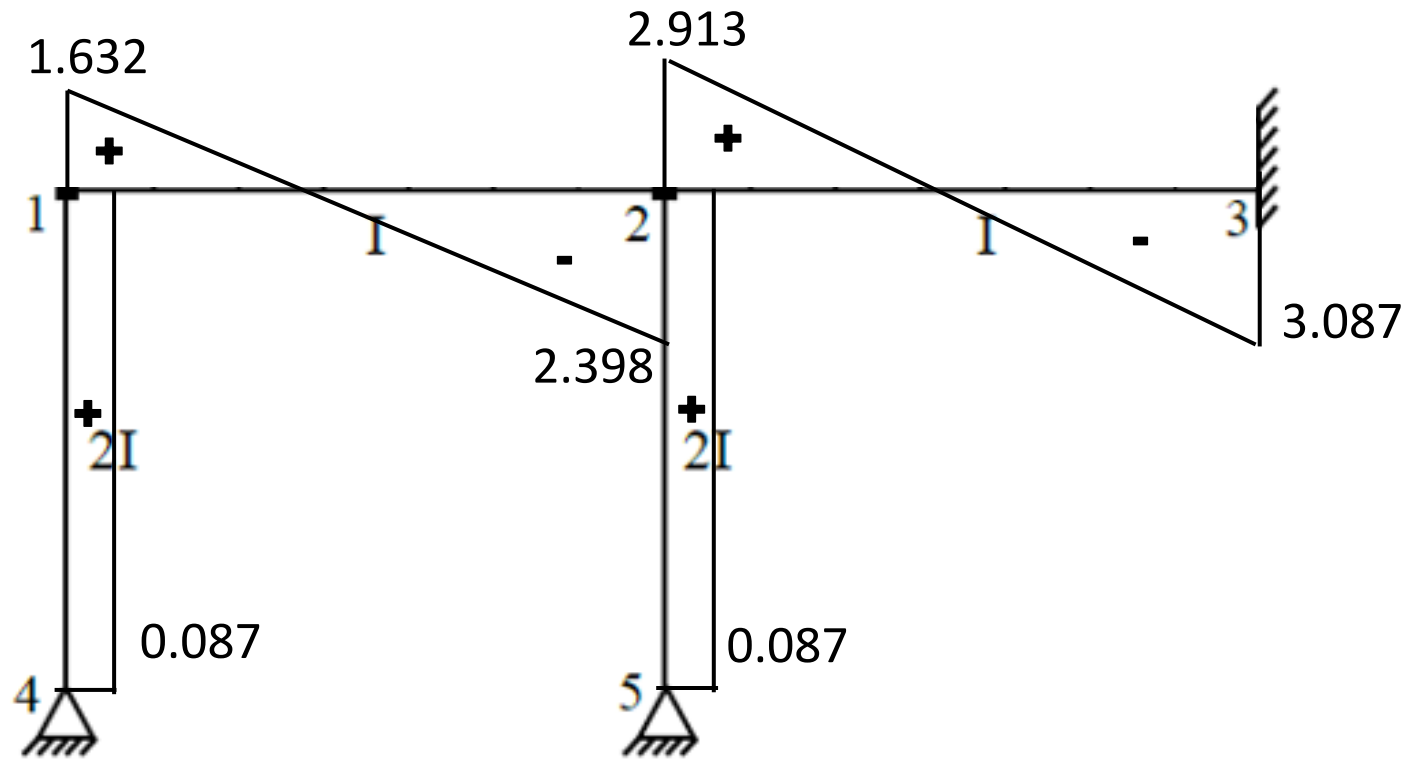
	(14)	(12)	$\longleftrightarrow 1/2$	(21)	(25)	(23)	$\longrightarrow 1/2$	
	0.50	0.50		0.375	0.375	0.25		
		0.667		-0.667		1.5		-1.5
		-0.156	\longleftarrow	-0.312	-0.312	-0.208	\longrightarrow	-0.104
	-0.255	-0.255	\longrightarrow	-0.128				
		0.024	\longleftarrow	0.048	0.048	0.032	\longrightarrow	0.016
	-0.012	-0.012	\longrightarrow	-0.006				
				0.002	0.002	0.0015	\longrightarrow	0.000
	-0.267	0.268		-1.063	-0.262	1.325		-1.588



Moment diyagramı



Kesme kuvveti diyagramı



CERCEVE

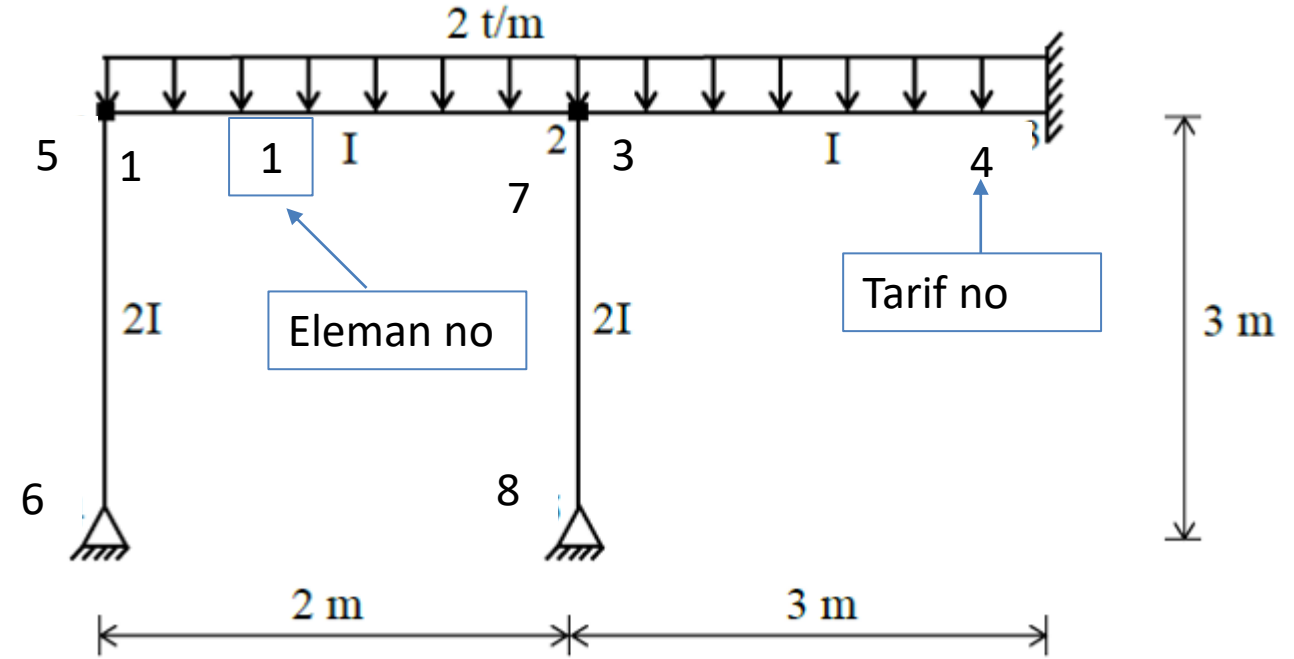
VERI DEGERLERI:

ELEMAN	BOY	ATALET MOM	DAGITMA KATS
1	2.00	1.000	0.500 0.375
2	3.00	1.000	0.250 0.000
3	3.00	2.000	0.500 1.000
4	3.00	2.000	0.375 1.000

YUKLEME NO= 1

ITERASYON SAYISI=10

ELEMAN	TARIF NO	ANKAS.UC MOM.	BAS.KI.KESME K.
1	1	-0.67	-2.00
	2	0.67	2.00
2	3	-1.50	-3.00
	4	1.50	3.00
3	5	0.00	0.00
	6	0.00	0.00
4	7	0.00	0.00
	8	0.00	0.00



HESAPLANAN DEGERLER:

*** E L E M A N U C K U V V E T L E R I ***

ELEMAN	TARIF NO	MOMENT	KESME KUVVETI
1	1	-0.268	-1.602
	2	1.063	2.398
2	3	-1.325	-2.913
	4	1.587	3.087
3	5	0.268	0.089
	6	0.000	0.089
4	7	0.262	0.087
	8	0.000	0.087

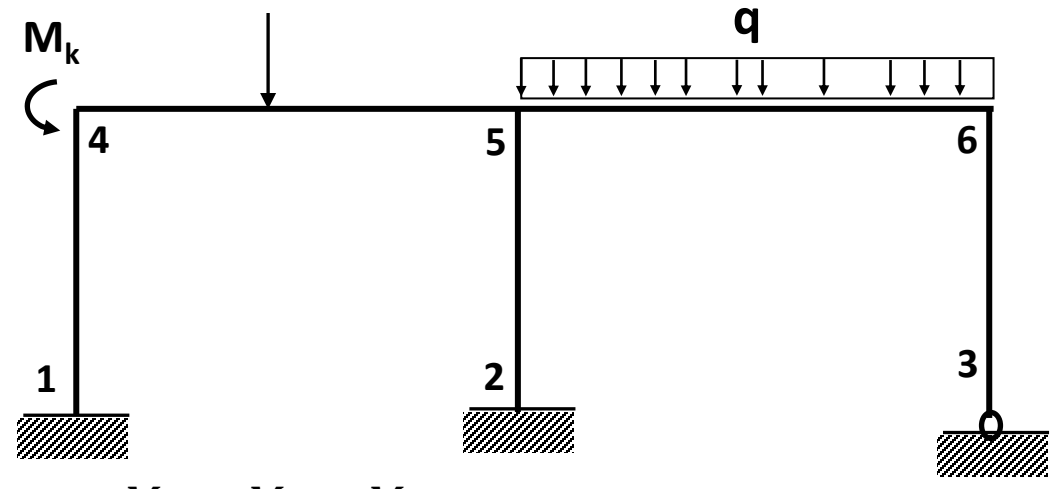
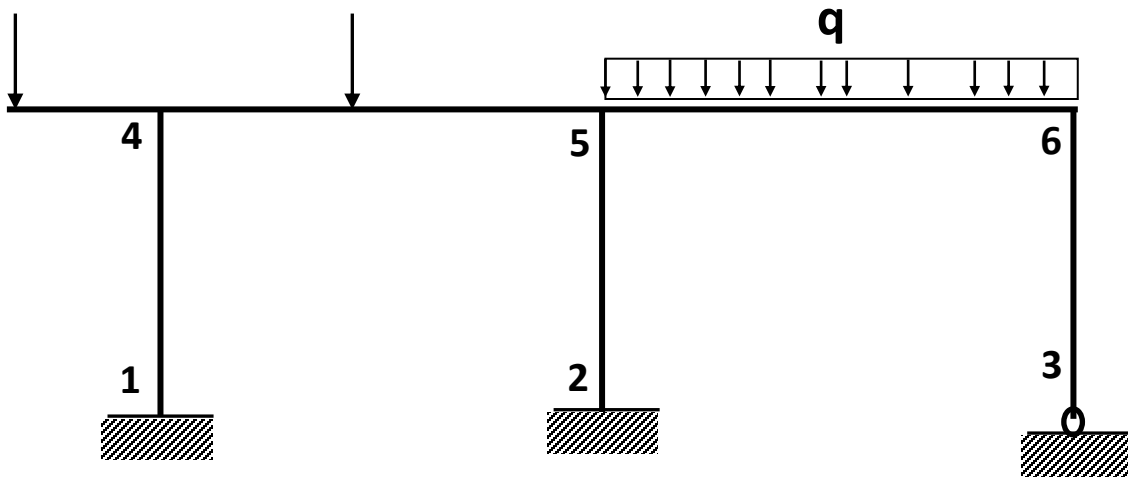
Xmax(i ucundan)= 0.801

1INCI ELEMAN ACIKLIK MOMENTI= 0.374

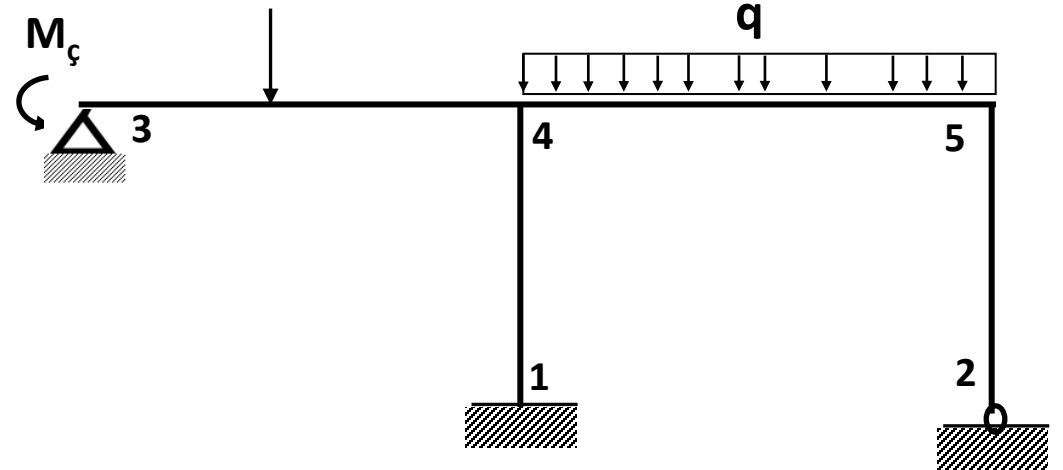
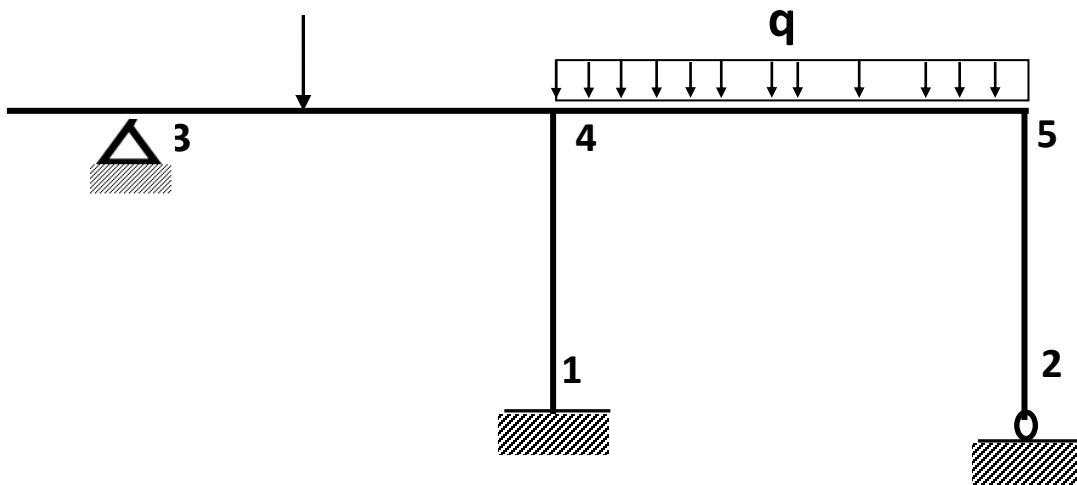
Xmax(i ucundan)= 1.456

2INCI ELEMAN ACIKLIK MOMENTI= 0.796

KONSOLLU SİSTEMLER

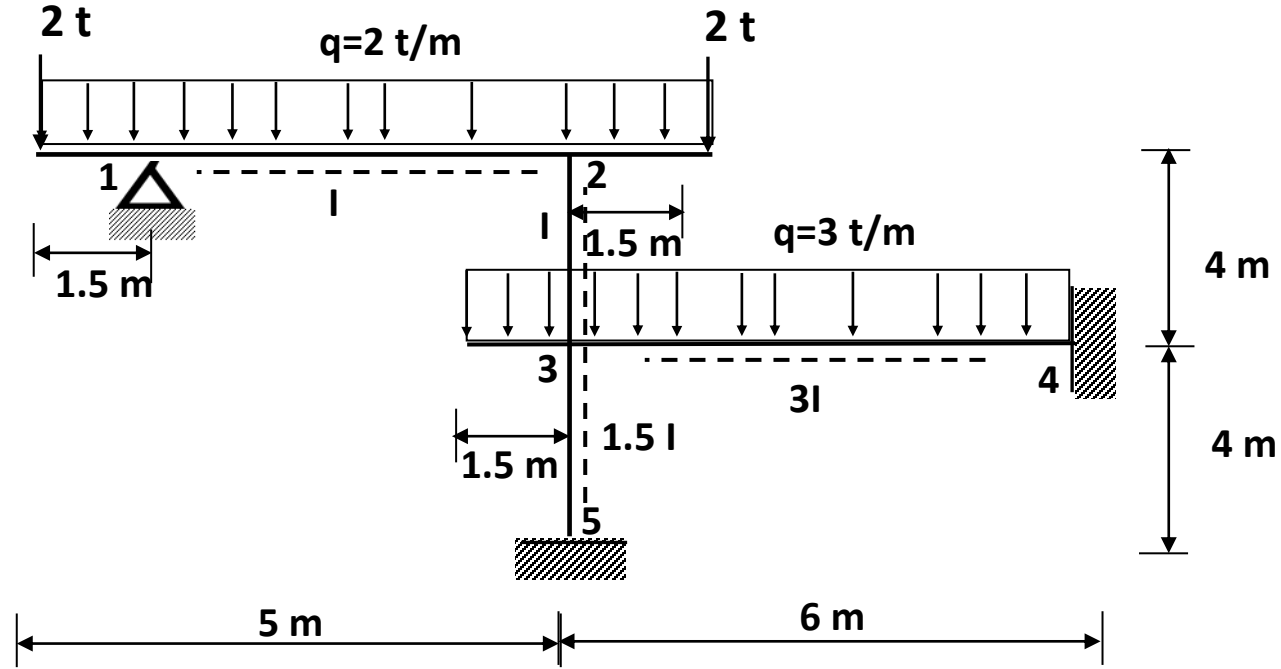


$$M_4 = M_k + M_{45}$$

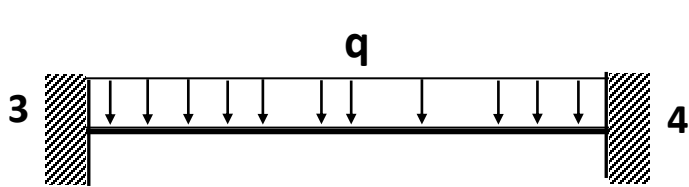


$$\overline{m}_{4\theta_4}^{43} = \frac{3EI}{L} \quad M_4 = \overline{\mathcal{M}}_{43} + \frac{M_\zeta}{2} + M_{45}$$

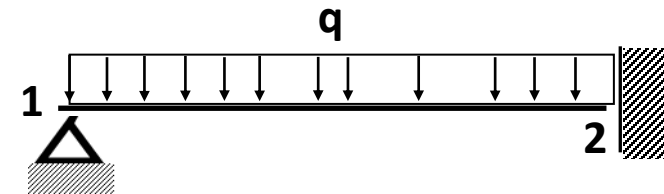
UYGULAMA 1



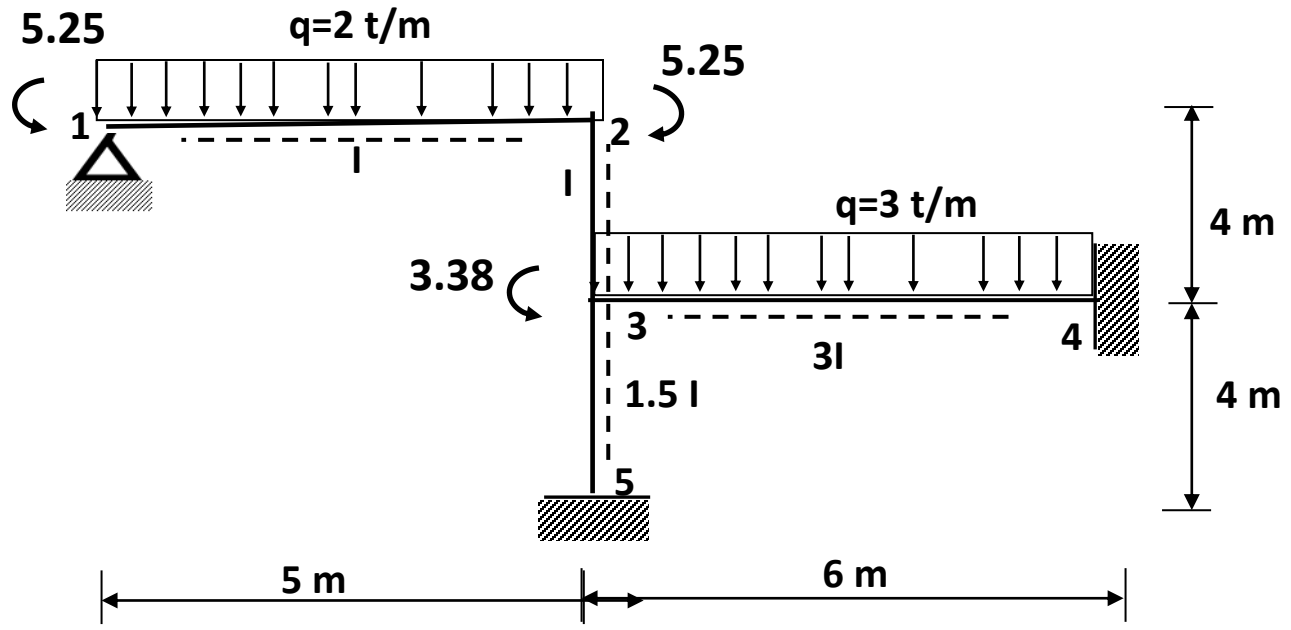
Cross metodunu kullanarak M N T diyagramlarını çiziniz.



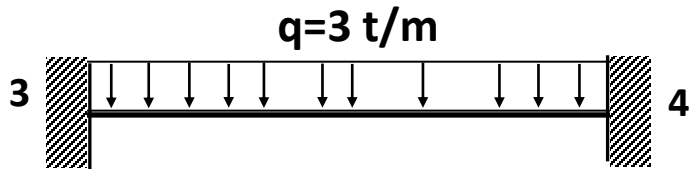
$$\mathcal{M}_{34} = -\mathcal{M}_{43} = \frac{qL^2}{12} = \frac{3 * 6^2}{12} = 9 \text{ tm}$$



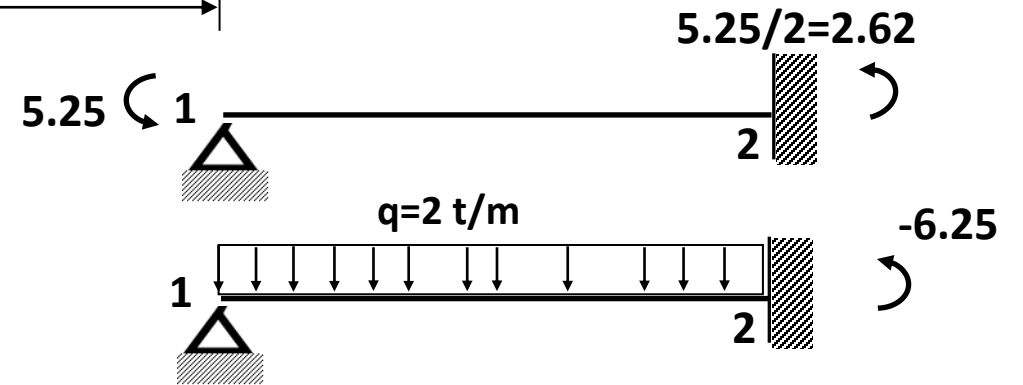
$$\mathcal{M}_{21} = -\frac{qL^2}{8} = -\frac{2 * 5^2}{8} = -6.25 \text{ tm}$$



1. Ankastrelilik uç momentleri:



$$\mathcal{M}_{34} = -\mathcal{M}_{43} = \frac{qL^2}{12} = \frac{3 * 6^2}{12} = 9 \text{ tm}$$



$$\overline{\mathcal{M}}_{21} = -\frac{qL^2}{8} = -\frac{2 * 5^2}{8} = -6.25 \text{ tm}$$

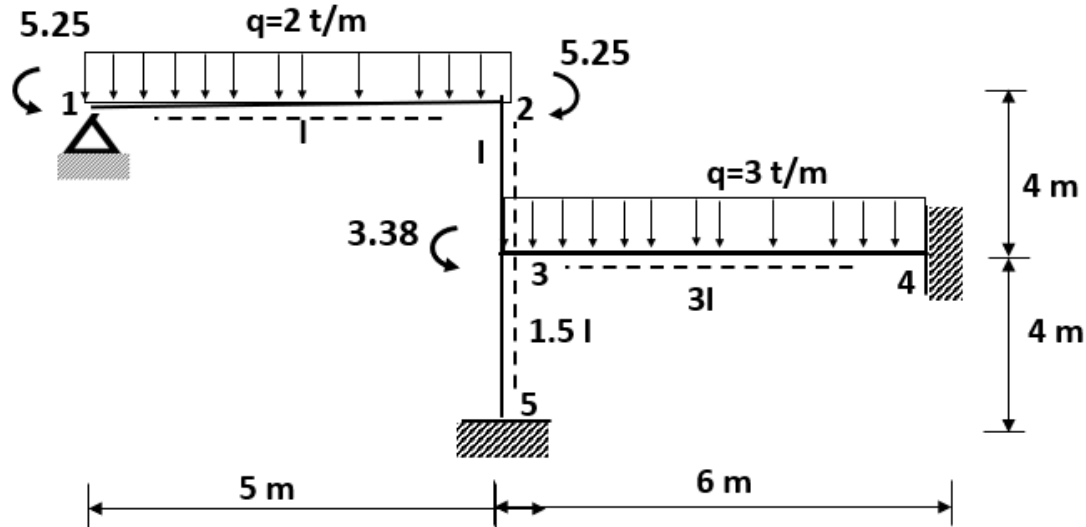
$$= \frac{+2.62}{-3.63}$$

2. Dağıtma sayıları :

$$r_{21} = \frac{m_{2\theta_2}^{21}}{m_{2\theta_2}^{21} + m_{2\theta_2}^{23}} = \frac{\frac{3EI}{5}}{\frac{3EI}{5} + \frac{4EI}{4}} = \frac{0.6}{0.6 + 1.0} = 0.375$$

$$r_{23} = \frac{m_{2\theta_2}^{23}}{m_{2\theta_2}^{23} + m_{2\theta_2}^{21}} = \frac{\frac{4EI}{4}}{\frac{4EI}{4} + \frac{3EI}{5}} = \frac{1}{1.0 + 0.6} = 0.625$$

$$\sum r = 1 \text{ olmalı} \rightarrow 0.375 + 0.625 = 1 \quad \checkmark$$



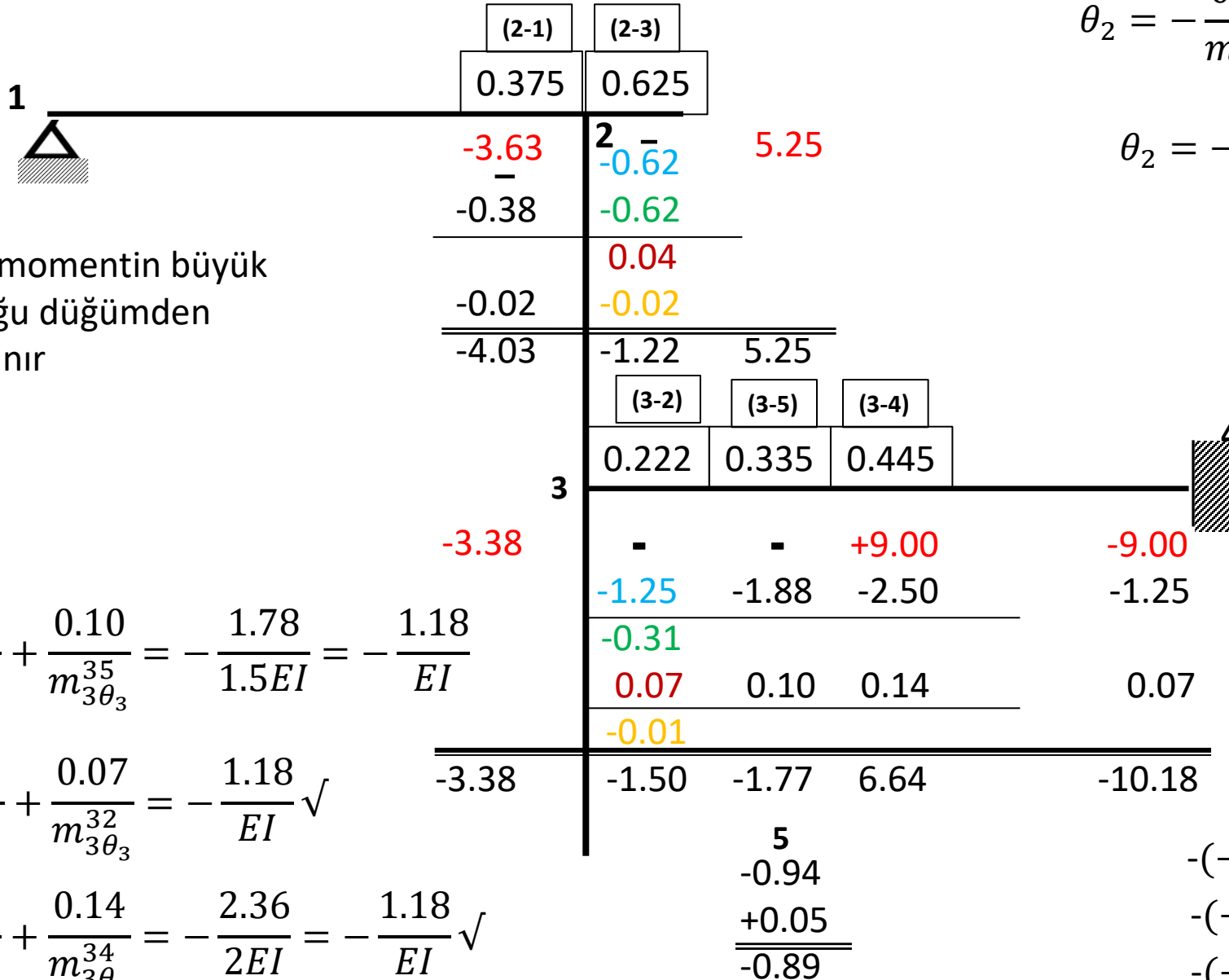
$$r_{32} = \frac{\frac{4EI}{4}}{\frac{4EI}{4} + \frac{4E3I}{6} + \frac{4E1.5I}{4}} = 0.222$$

$$r_{34} = \frac{\frac{4E3I}{6}}{\frac{4EI}{4} + \frac{4E3I}{6} + \frac{4E1.5I}{4}} = 0.445$$

$$r_{35} = \frac{\frac{4E1.5I}{6}}{\frac{4EI}{4} + \frac{4E3I}{6} + \frac{4E1.5I}{4}} = 0.333$$

$$\sum r = 1 \text{ olmalı} \rightarrow 0.222 + 0.445 + 0.333 = 1 \quad \checkmark$$

3. Dengeleme :



Fark momentin büyük olduğu düğümden başlanır

$$\theta_3 = -\frac{1.88}{m_{3\theta_3}^{35}} + \frac{0.10}{m_{3\theta_3}^{35}} = -\frac{1.78}{1.5EI} = -\frac{1.18}{EI}$$

$$\theta_3 = -\frac{1.25}{m_{3\theta_3}^{32}} + \frac{0.07}{m_{3\theta_3}^{32}} = -\frac{1.18}{EI} \sqrt{\quad}$$

$$\theta_3 = -\frac{2.5}{m_{3\theta_3}^{34}} + \frac{0.14}{m_{3\theta_3}^{34}} = -\frac{2.36}{2EI} = -\frac{1.18}{EI} \sqrt{\quad}$$

$$\theta_2 = -\frac{0.38}{m_{2\theta_2}^{21}} - \frac{0.02}{m_{2\theta_2}^{21}} = -\frac{0.40}{0.6EI} = -\frac{0.66}{EI}$$

$$\theta_2 = -\frac{0.62}{m_{2\theta_2}^{23}} - \frac{0.02}{m_{2\theta_2}^{23}} = -\frac{0.64}{EI} \approx \sqrt{\quad}$$

3 düğümü

$$-3.38 + 9 = 5.62$$

$$-5.62 * 0.222 = -1.25$$

$$-5.62 * 0.335 = -1.88$$

$$-5.62 * 0.445 = -2.50$$

2 düğümü

$$-3.63 + 5.25 - 0.62 = 1.0$$

$$-1.0 * 0.375 = -0.38$$

$$-1.0 * 0.625 = -0.62$$

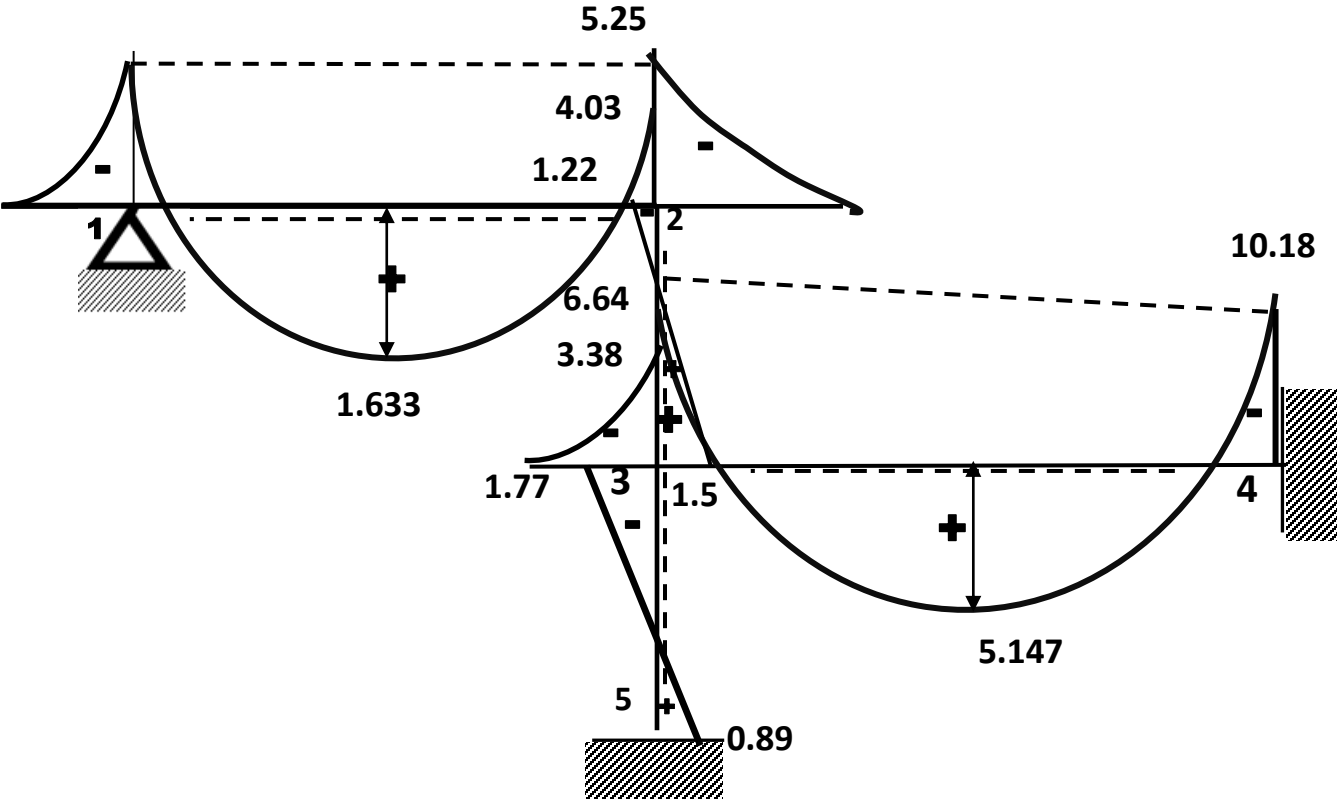
3 düğümü

$$-(-0.31) * 0.222 = 0.07$$

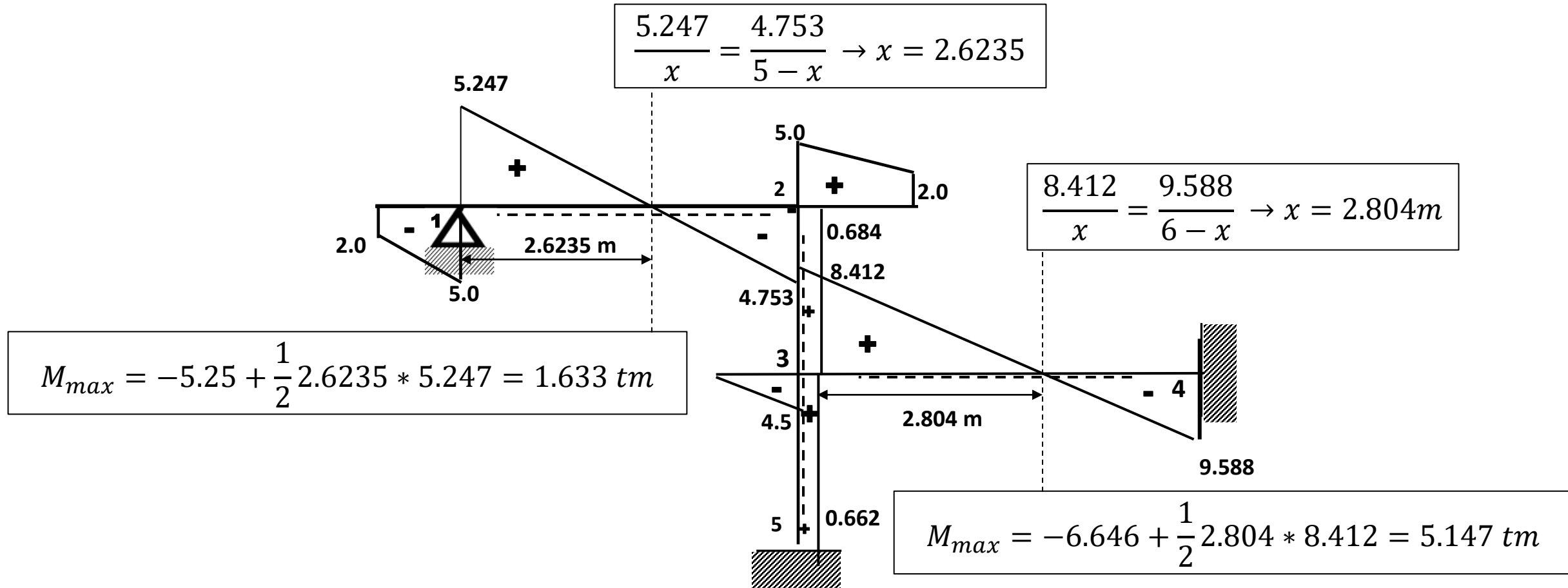
$$-(-0.31) * 0.335 = 0.10$$

$$-(-0.31) * 0.445 = 0.14$$

4. Moment diyagramı:



5. Kesme kuvveti diyagramı:



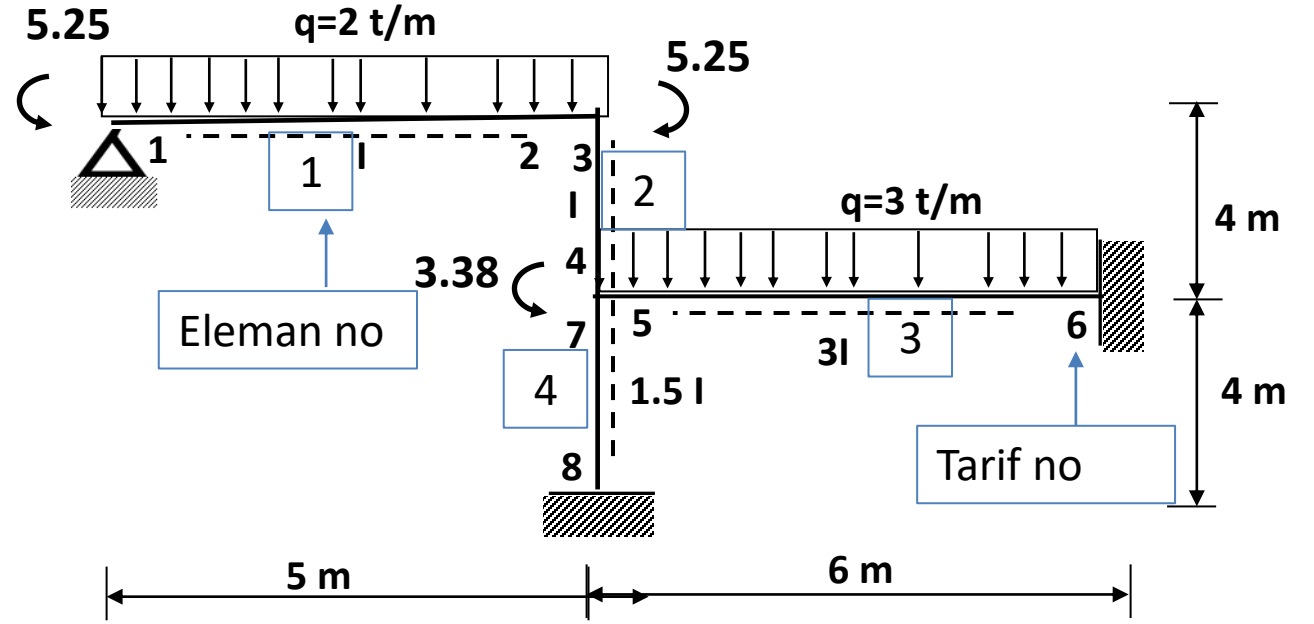
V E R I D E G E R L E R I :

ELEMAN	BOY	ATALET MOM	DAGITMA KATS
1	5.00	1.000	1.000 0.375
2	4.00	1.000	0.625 0.222
3	6.00	3.000	0.444 0.000
4	4.00	1.500	0.333 0.000

YUKLEME NO= 1

ITERASYON SAYISI=10

ELEMAN	TARIF NO	ANKAS.UC MOM.	BAS.KI.KESME K.
1	1	1.08	-5.00
	2	-1.08	5.00
2	3	0.00	0.00
	4	0.00	0.00
3	5	-5.62	-9.00
	6	9.00	9.00
4	7	0.00	0.00
	8	0.00	0.00



HESAPLANAN DEGERLER:

*** E L E M A N U C K U V V E T L E R I ***

ELEMAN	TARIF NO	MOMENT	KESME KUVVETI
1	1	-5.250	-5.247
	2	4.014	4.753
2	3	1.236	0.684
	4	1.501	0.684
3	5	-6.646	-8.412
	6	10.177	9.588
4	7	1.765	0.662
	8	0.883	0.662

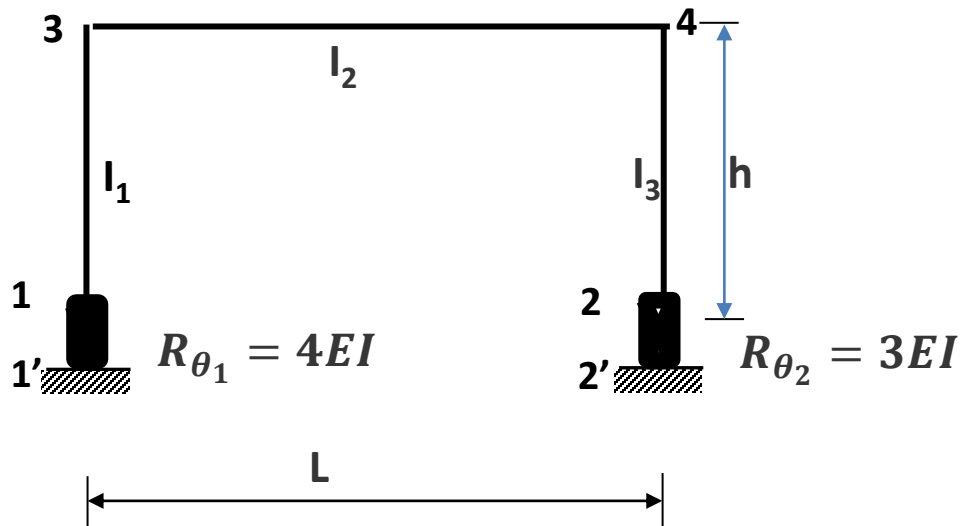
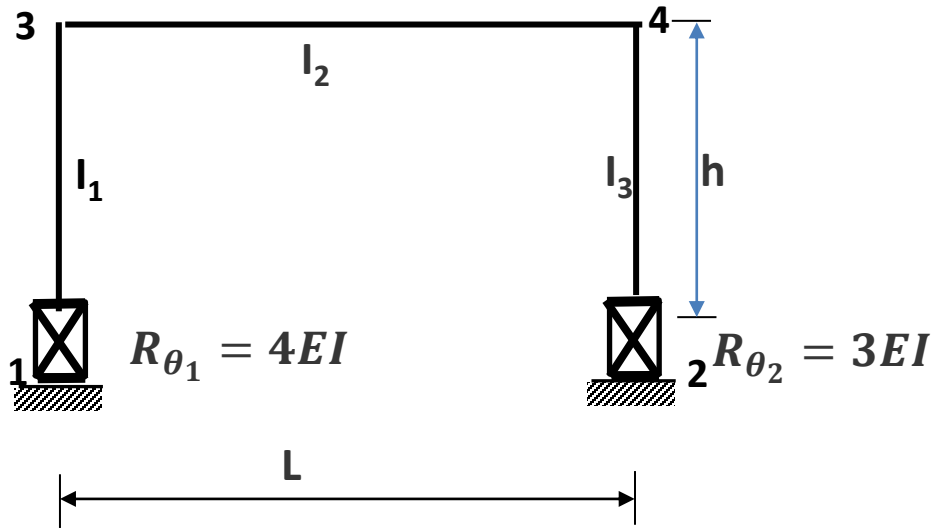
Xmax(i ucundan)= 2.624

1INCI ELEMAN ACIKLIK MOMENTI= 1.633

Xmax(i ucundan)= 2.804

3INCI ELEMAN ACIKLIK MOMENTI= 5.146

ELASTİK MESNETLİ SİSTEMLER

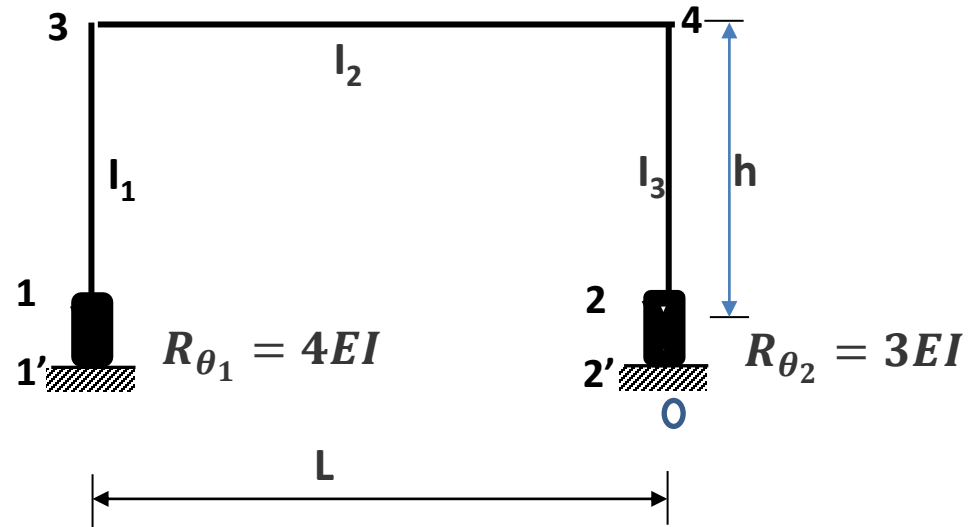


$$\frac{M}{\theta} = R_{\theta} \quad \theta = 1 \quad m_{i\theta_i} = M = R_{\theta}$$

$$r_{13} = \frac{m_{1\theta_1}^{13}}{m_{1\theta_1}^{13} + m_{1\theta_1}^{11'}} = \frac{\frac{4EI_1}{h}}{\frac{4EI_1}{h} + 4EI}$$

$$r_{11'} = \frac{m_{1\theta_1}^{11'}}{m_{1\theta_1}^{13} + m_{1\theta_1}^{11'}} = \frac{4EI}{\frac{4EI_1}{h} + 4EI}$$

$$m_{3\theta_3}^{31} = \frac{4EI_1}{h}$$



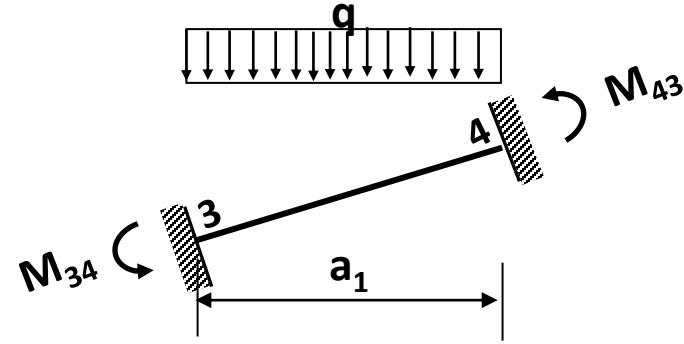
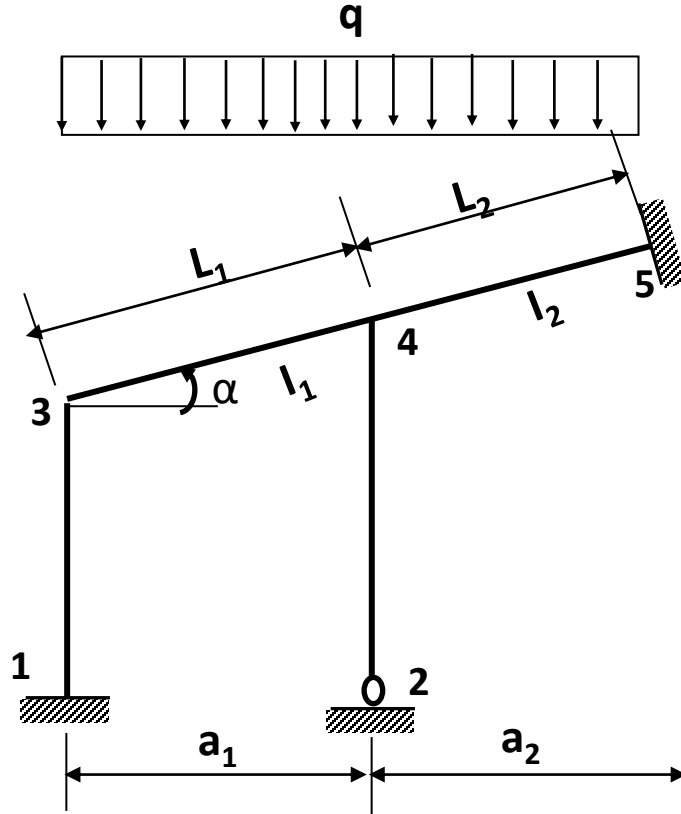
$$m_{3\theta_3}^{31} = \frac{4EI_1}{h}$$

$$r_{24} = \frac{m_{22}^{24}}{m_{2\theta_2}^{24} + m_{2\theta_2}^{22'}} = \frac{\frac{4EI_3}{h}}{\frac{4EI_3}{h} + 3EI}$$

$$r_{22'} = \frac{m_{2\theta_2}^{22'}}{m_{2\theta_2}^{24} + m_{2\theta_2}^{22'}} = \frac{3EI}{\frac{4EI_3}{h} + 3EI}$$

$$m_{4\theta_4}^{42} = \frac{4EI_3}{h}$$

Çubukları yatay ile açı yapan sistemler:



$$\mathcal{M}_{34} = -\mathcal{M}_{43} = \frac{q a_1^2}{12}$$

$$\mathcal{M}_{43} = -\mathcal{M}_{45} = \frac{q a_2^2}{12}$$

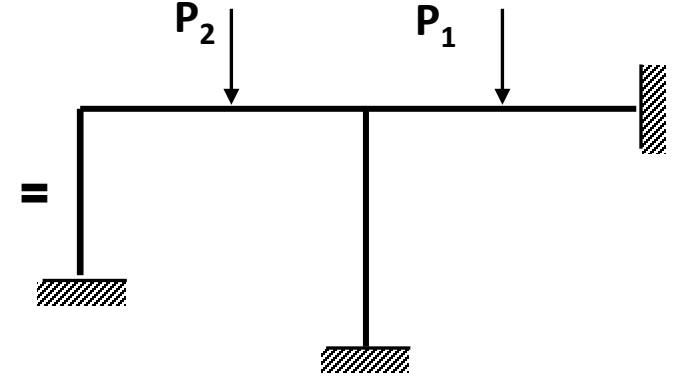
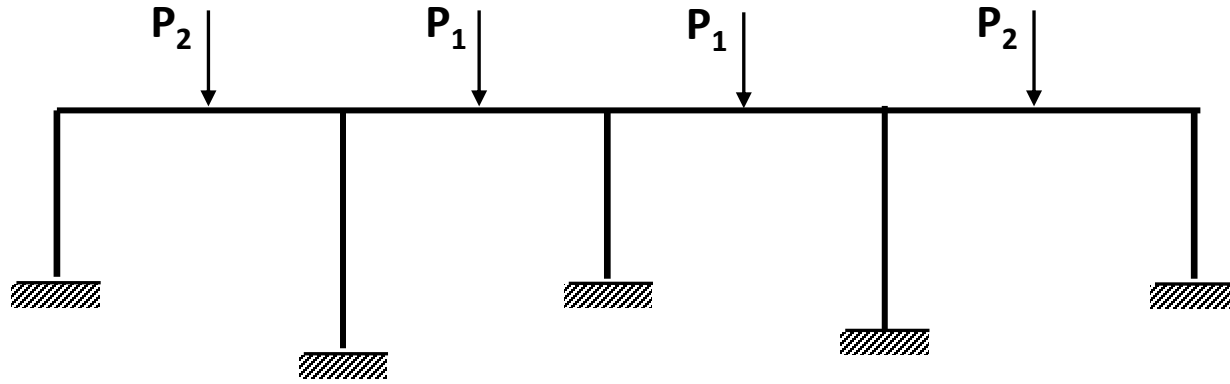
$$m_{3\theta_3}^{34} = \frac{4EI_1}{L_1} \quad m_{4\theta_{34}}^{45} = \frac{4EI_2}{L_2}$$

Ankastrelık momentleri hesaplanırken yatay mesafe alınır. Birim deplasman sabitleri hesaplanırken çubuk boyu göz önüne alınır.

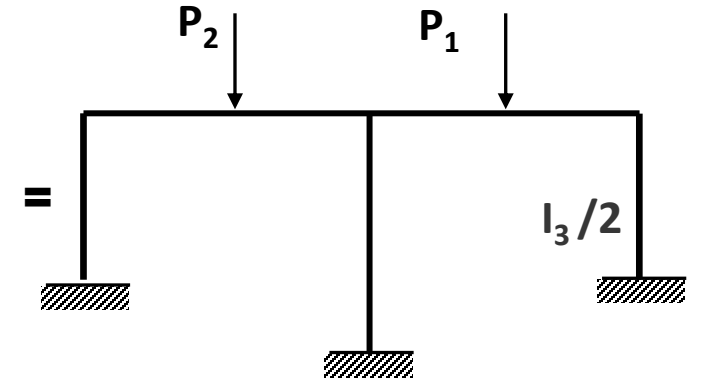
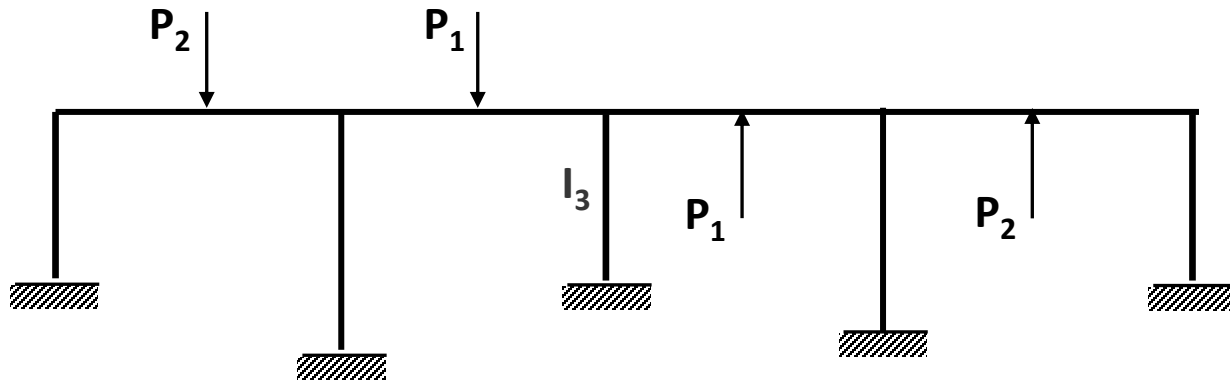
SİMETRİK VE ANTİMETRİK SİSTEMLER

1. Simetri ekseninde kolon bulunan sistemler

1.1 Simetrik yük hali

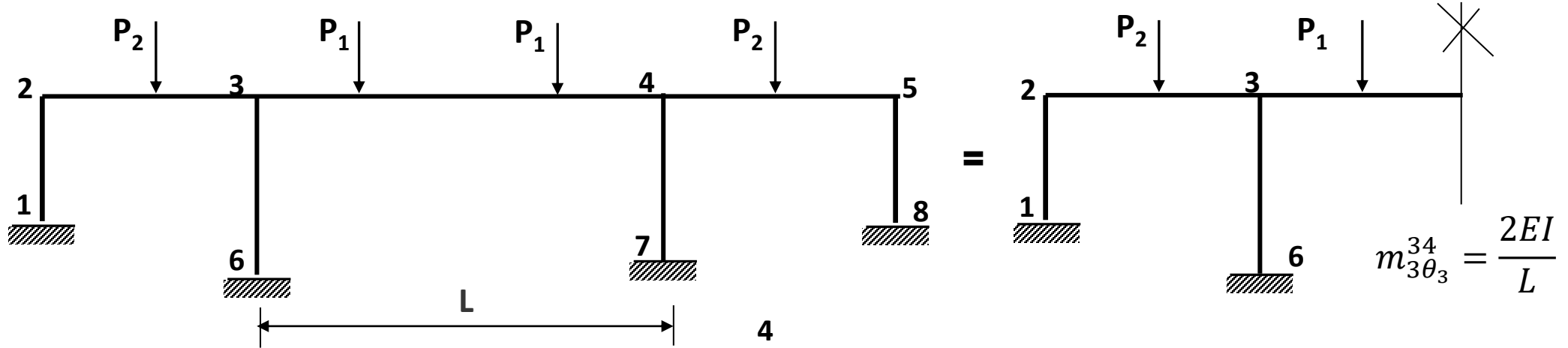


1.2 Antimetrik yükleme hali

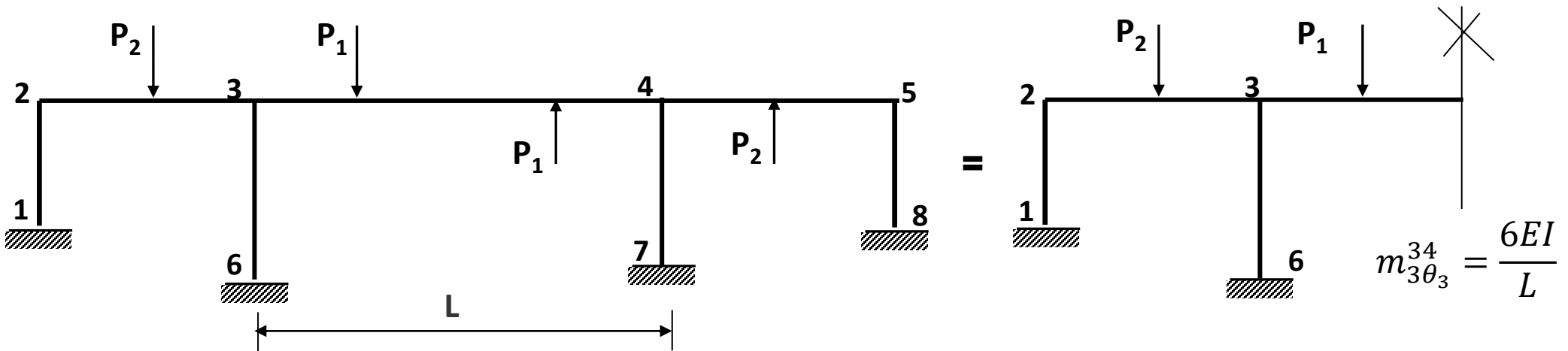


2. Simetri ekseninde kolon veya düğüm noktası bulunmayan elemanlar

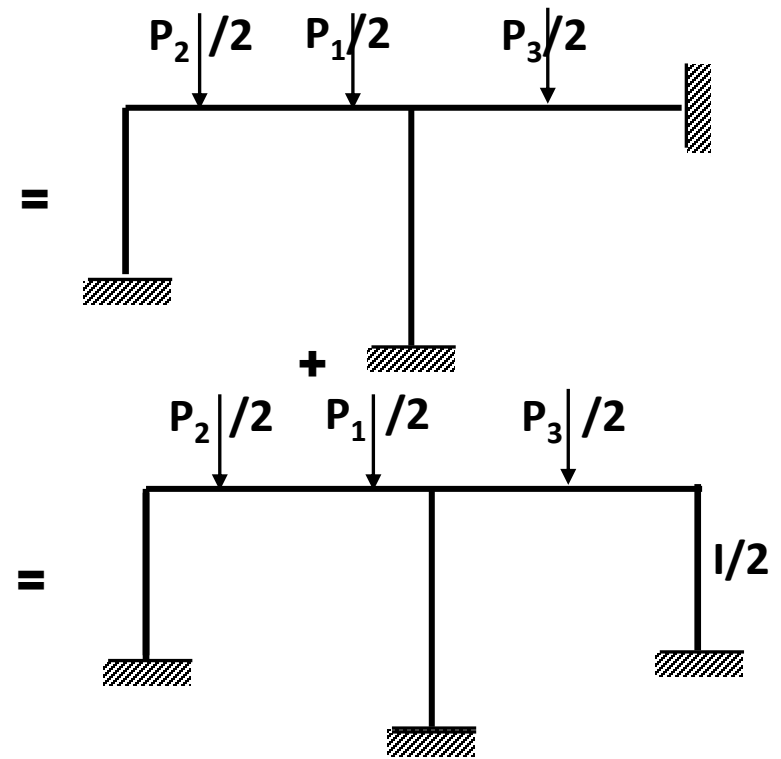
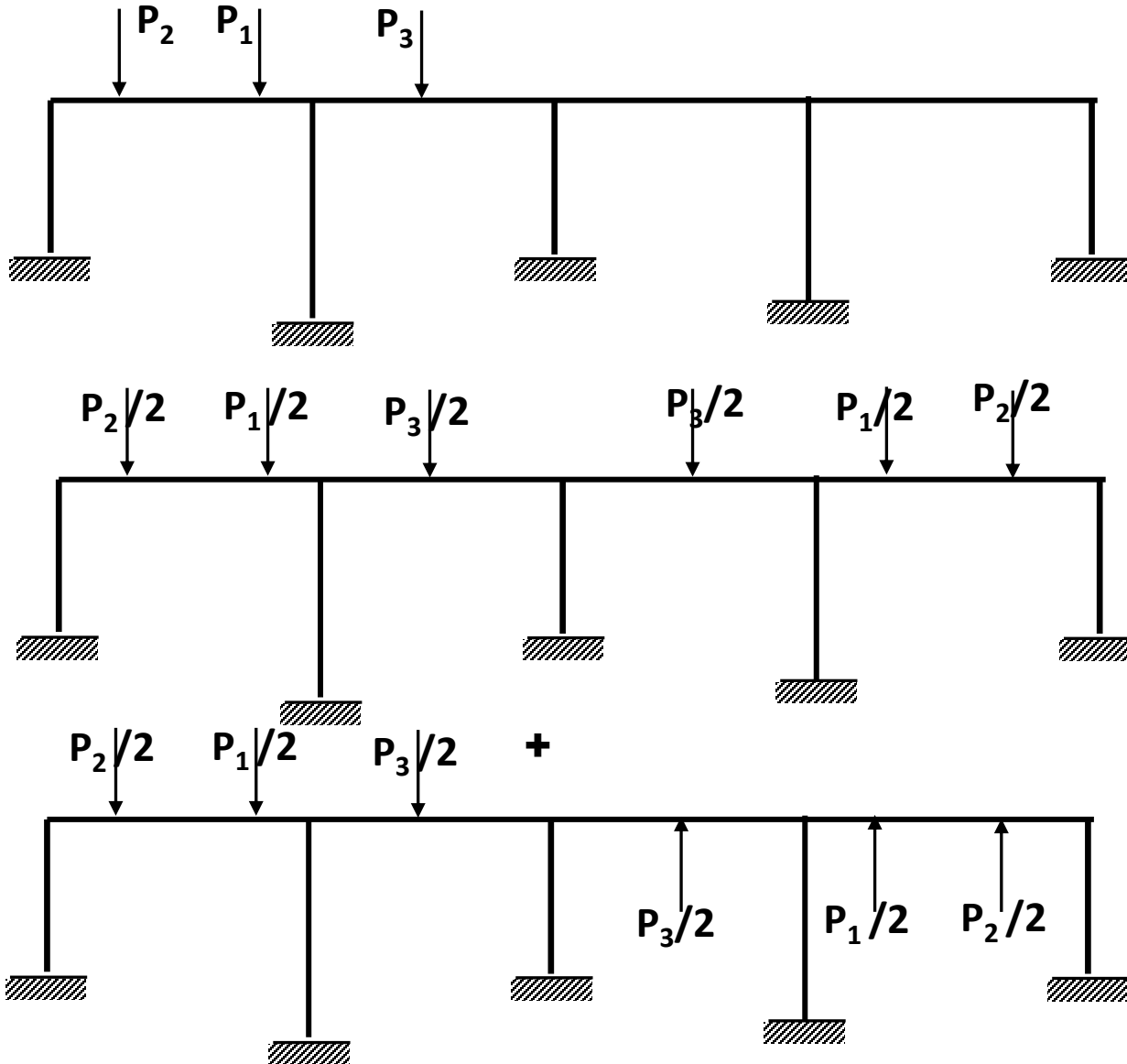
2.1 Simetrik yük hali



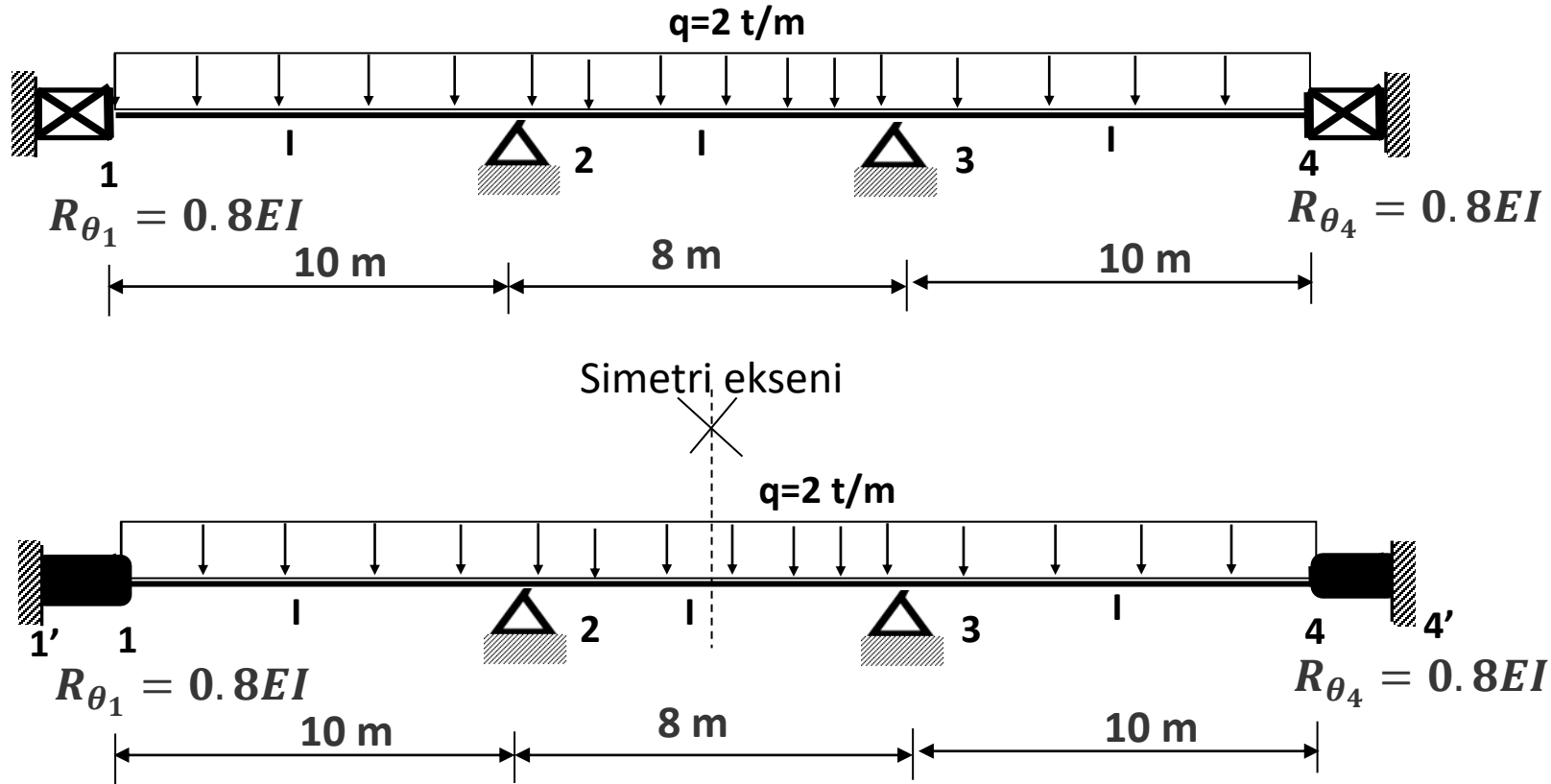
2.2 Antimetrik yük hali



3. Gelişi güzel yüklü simetrik sistemler



UYGULAMA 2



1. Ankastrelik momentleri:

$$\mathcal{M}_{12} = -\mathcal{M}_{21} = \frac{2 * 10^2}{12} = 16.67 \text{ tm} \quad \mathcal{M}_{23} = -\mathcal{M}_{32} = \frac{2 * 8^2}{12} = 10.67 \text{ tm}$$

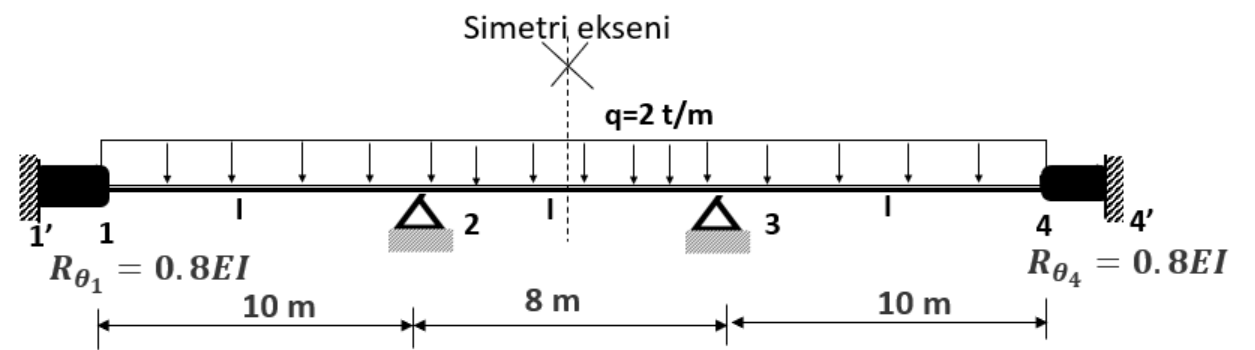
$$\mathcal{M}_{34} = -\mathcal{M}_{43} = \frac{2 * 10^2}{12} = 16.67 \text{ tm}$$

2. Dağıtma sayıları:

$$r_{11'} = \frac{0.8EI}{0.8EI + \frac{4EI}{10}} = 0.667 \quad r_{12} = \frac{\frac{4EI}{10}}{0.8EI + \frac{4EI}{10}} = 0.333$$

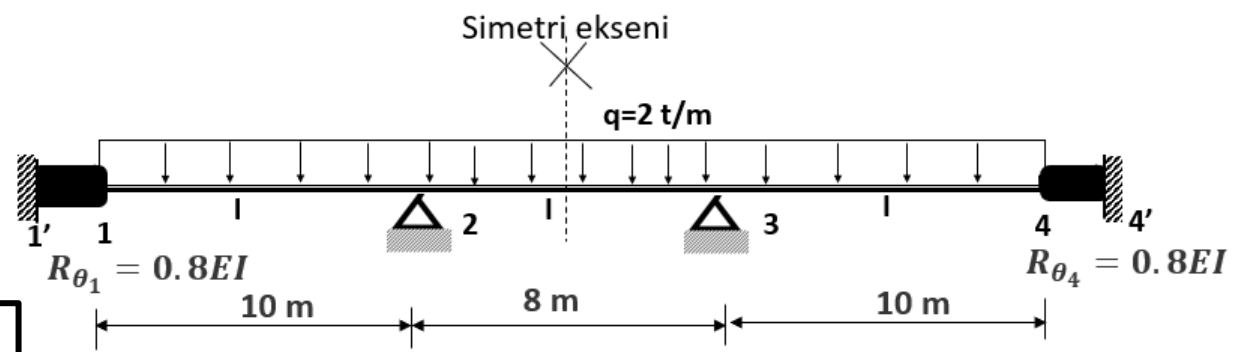
$$r_{21} = \frac{\frac{4EI}{10}}{\frac{4EI}{10} + \frac{2EI}{8}} = 0.615 \quad r_{23} = \frac{\frac{2EI}{8}}{\frac{4EI}{10} + \frac{2EI}{8}} = 0.385$$

Simetri özelliğinden yarısı alınır

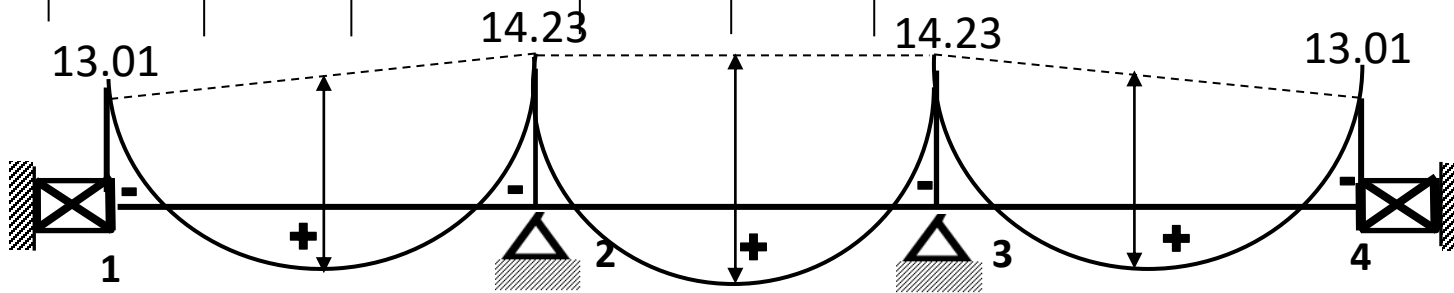


3. Cross dengelemesi:

(1-1')	(1-2)	$\frac{1}{2}$	(2-1)	(2-3)
0.667	0.333		0.615	0.385
	16.67		-16.67	10.67
-11.12	-5.65	→	-2.78	
	2.70	←	5.40	3.38
-1.80	-0.90	→	-0.45	
	0.14	←	0.28	0.17
-0.09	-0.05	→	-0.03	
-13.01	13.01		0.02	0.01
			-14.23	14.23



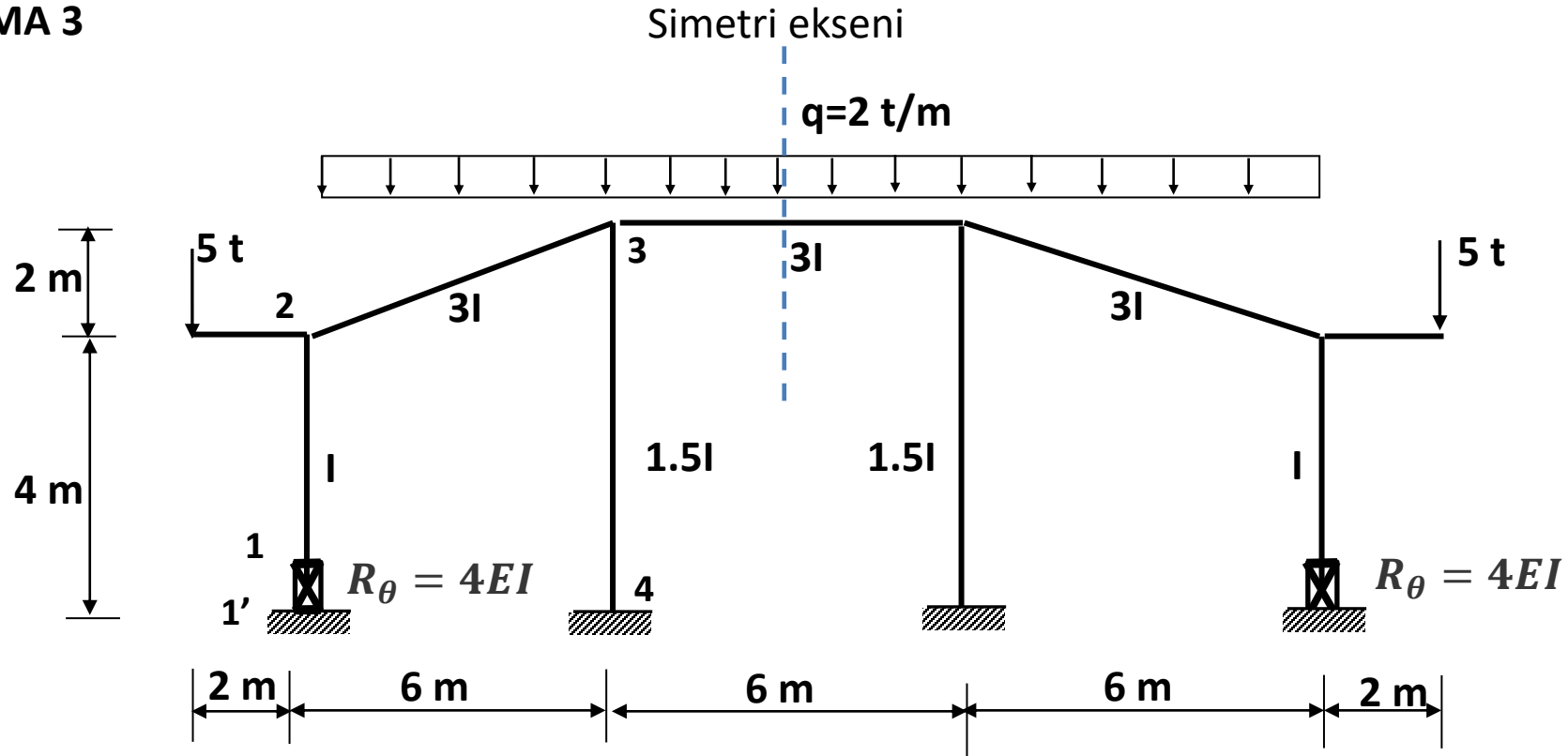
4. Moment diyagramı:



$$\frac{qL^2}{8} = \frac{2 * 8^2}{8} = 16 \text{ tm}$$

$$\frac{qL^2}{8} = \frac{2 * 10^2}{8} = 25 \text{ tm}$$

UYGULAMA 3



1. Ankastrelik momentleri:

$$\mathcal{M}_{23} = -\mathcal{M}_{32} = \frac{2 * 6^2}{12} = 6 \text{ tm} \quad \mathcal{M}_{33'} = \mathcal{M}_{3'3} - \frac{2 * 6^2}{12} = 6 \text{ tm} \quad M_{2_{konsol}} = -2 * 5 = -10 \text{ tm}$$

2. Birim deplasman sabitleri

$$m_{1\theta_1}^{12} = \frac{4EI}{L} = \frac{4EI}{4} = EI \quad m_{1\theta_1}^{11'} = 4EI \quad m_{2\theta_2}^{21} = \frac{4EI}{L} = \frac{4EI}{4} = EI \quad m_{2\theta_2}^{23} = \frac{4EI}{L} = \frac{4E3I}{6.32} = 1.9EI$$

$$m_{3\theta_3}^{32} = 1.9EI \quad m_{3\theta_3}^{33'} = \frac{2E3I}{6} = EI \quad m_{3\theta_3}^{34} = \frac{4E1.5I}{6} = EI$$

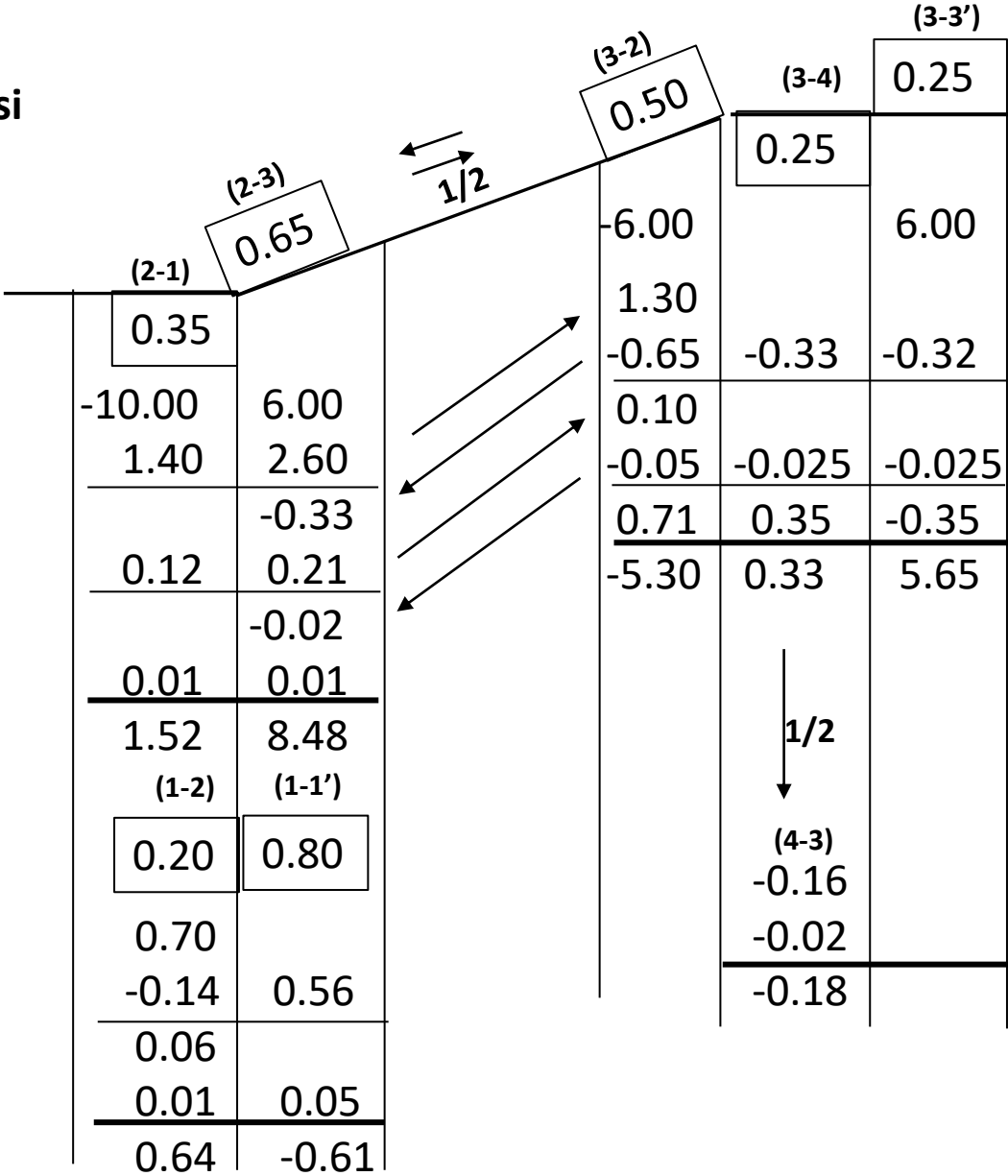
2. Dağıtma sayıları

$$r_{12} = \frac{EI}{EI + 4EI} = 0.20 \quad r_{11'} = \frac{4EI}{EI + 4EI} = 0.80 \quad r_{12} + r_{11'} = 1 \rightarrow 0.20 + 0.80 = 1.0 \checkmark$$

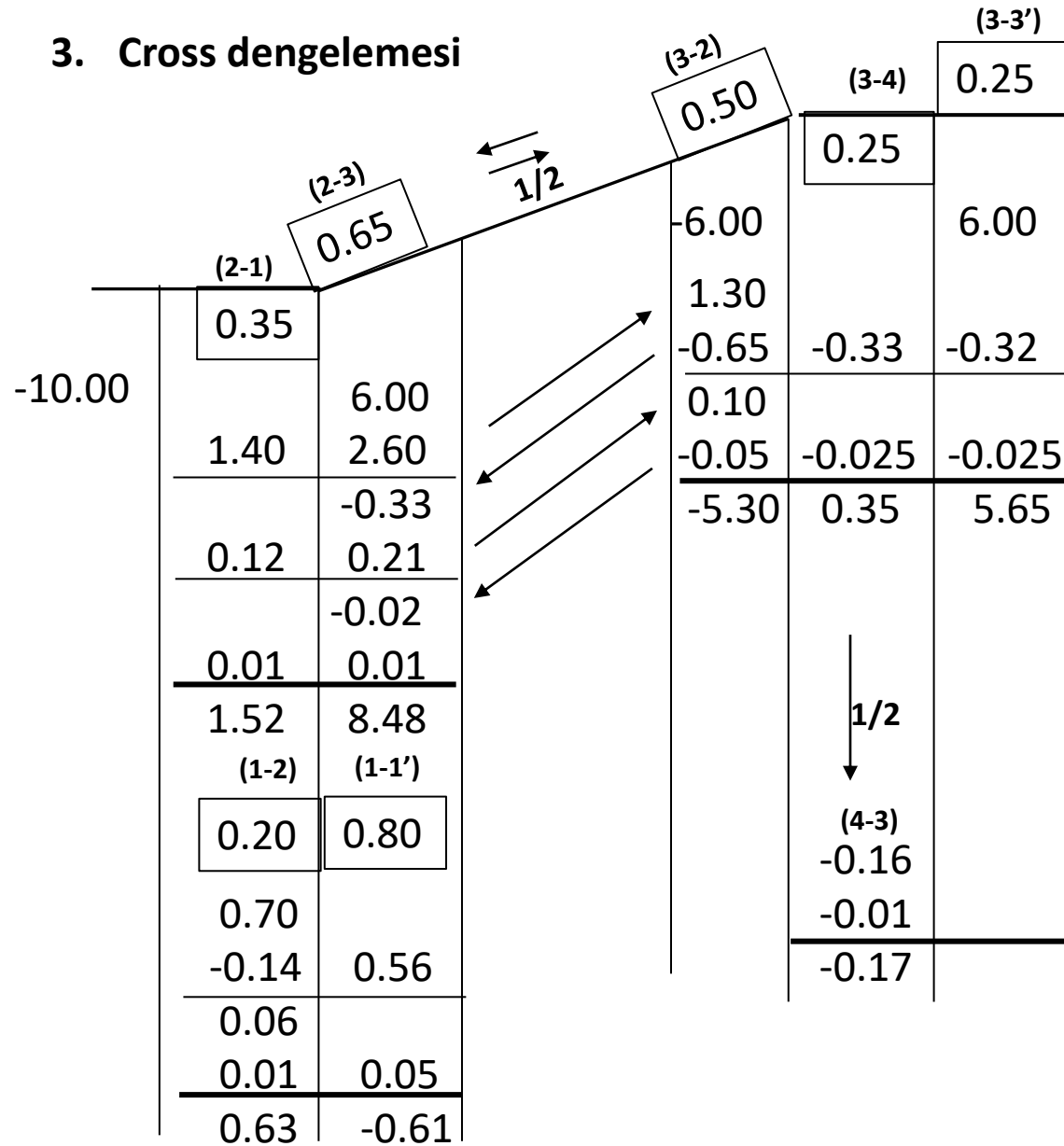
$$r_{21} = \frac{EI}{EI + 1.9EI} = 0.35 \quad r_{23} = \frac{1.9EI}{EI + 1.9EI} = 0.65 \quad r_{21} + r_{23} = 1 \rightarrow 0.35 + 0.65 = 1.0 \checkmark$$

$$r_{32} = \frac{1.9EI}{1.9EI + EI + EI} = 0.50 \quad r_{33'} = \frac{EI}{1.9EI + EI + EI} = 0.25 \quad r_{34} = 0.25 \quad r_{32} + r_{33'} + r_{34} = 1 \checkmark$$

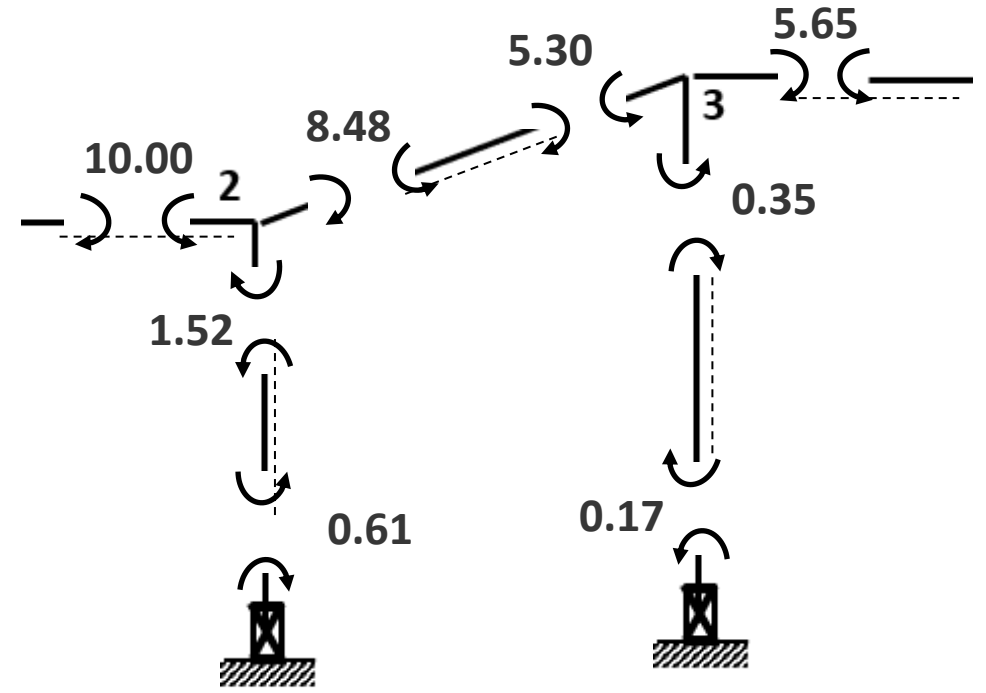
3. Cross dengelemesi



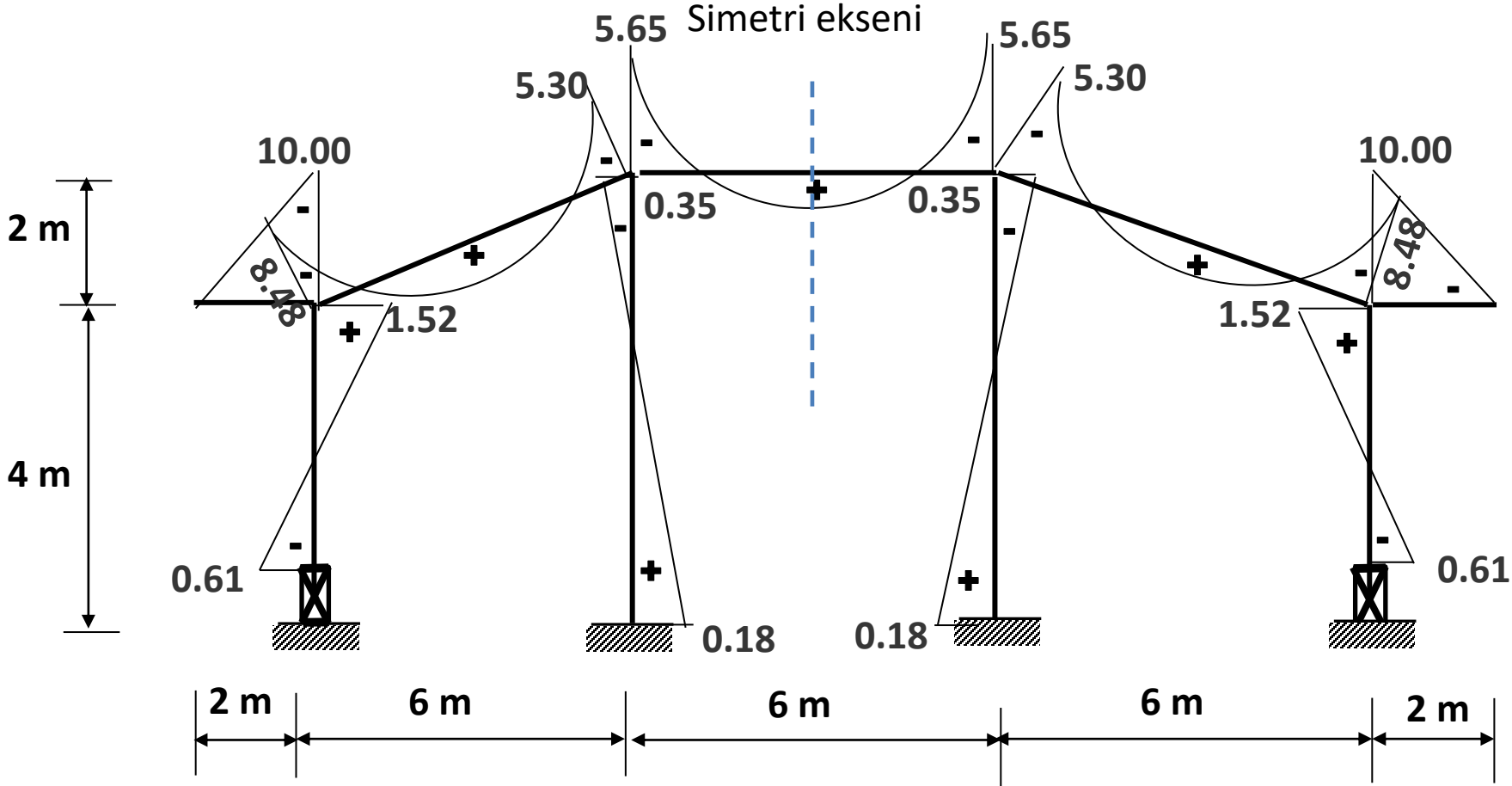
3. Cross dengelemesi



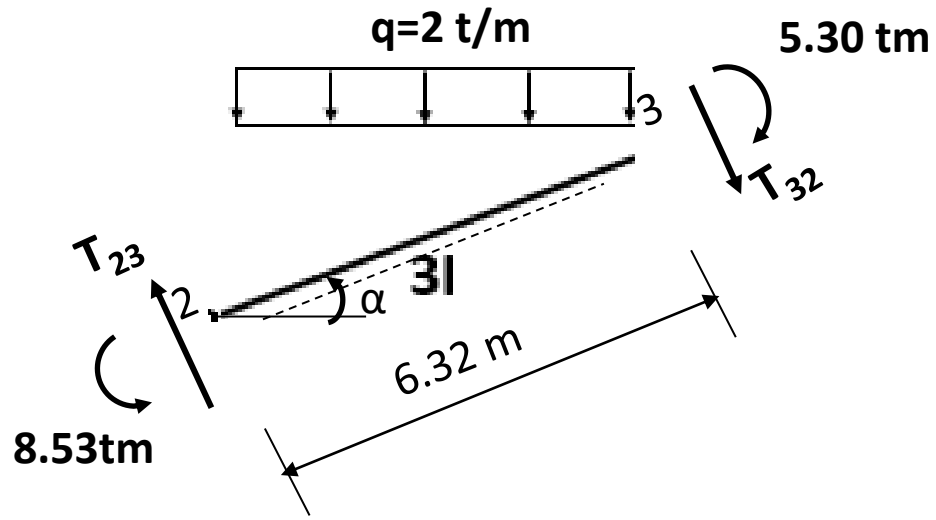
4. Dönüştürme



5. Moment diyagramı



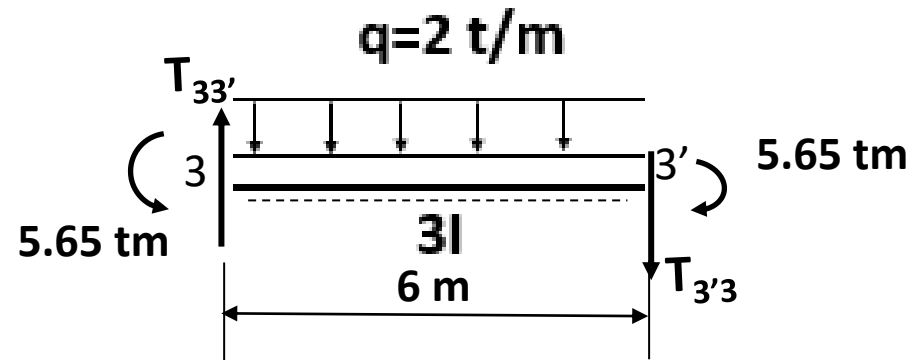
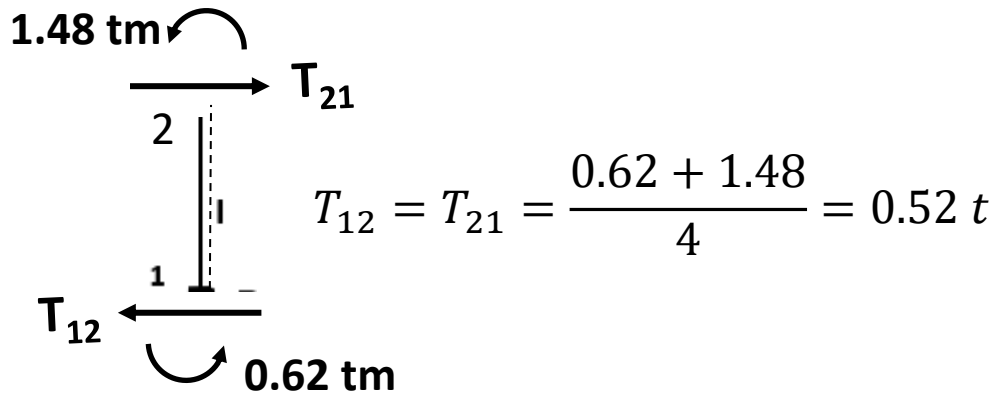
6. Kesme kuvvetlerinin hesabı



$$T_{konsol} = -5 t$$

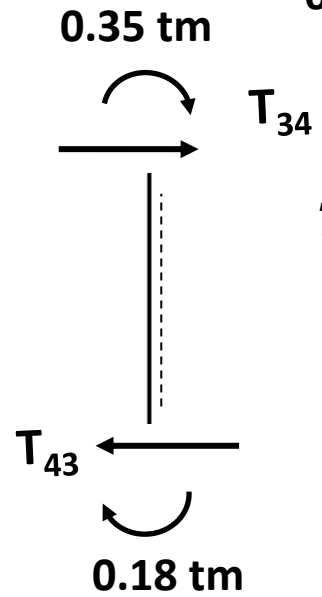
$$T_{23} = \frac{2 * 6}{2} \frac{6}{6.32} + \frac{8.53 - 5.30}{6.32} = 6.20 t$$

$$T_{23} = -\frac{2 * 6}{2} \frac{6}{6.32} + \frac{8.53 - 5.30}{6.32} = -5.18 t$$



$$T_{33'} = \frac{2 * 6}{2} + \frac{5.65 - 5.65}{6.0} = 6 t = -T_{3'3}$$

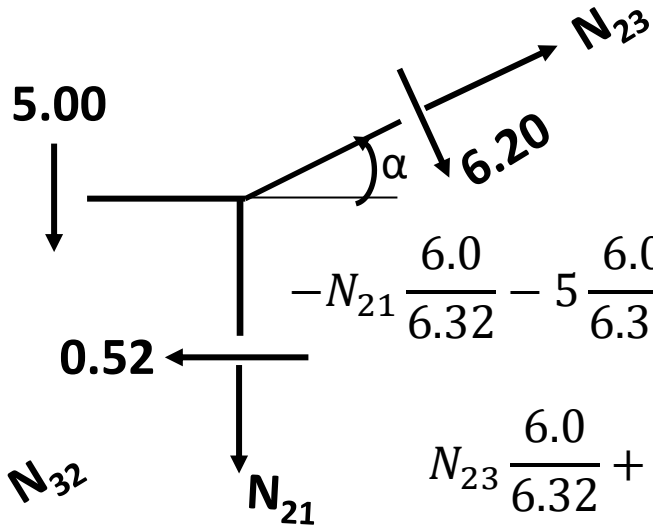
6. Kesme kuvvetlerinin hesabı



$$T_{34} = T_{43} = \frac{-0.18 - 0.35}{6} = -0.08 t$$

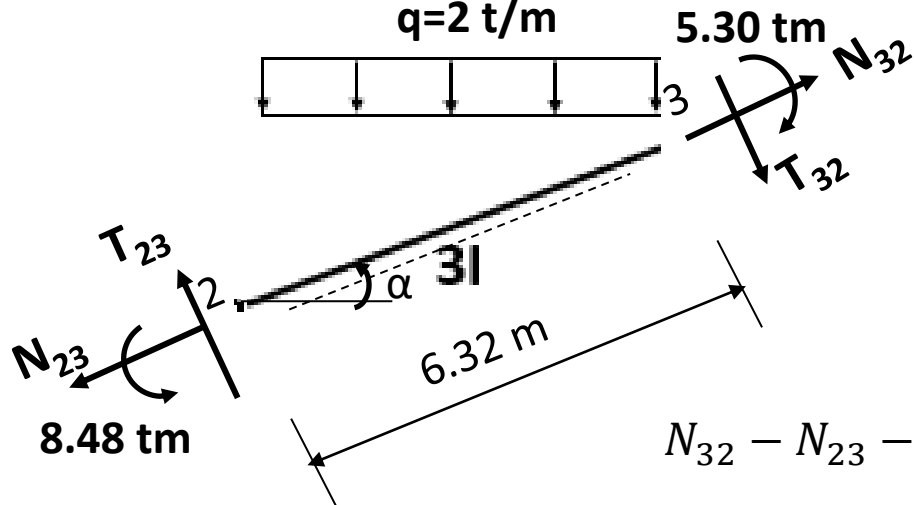
7. Normal kuvvetlerinin hesabı

$$\sin\alpha = \frac{2}{6.32} \quad \cos\alpha = \frac{6}{6.32}$$



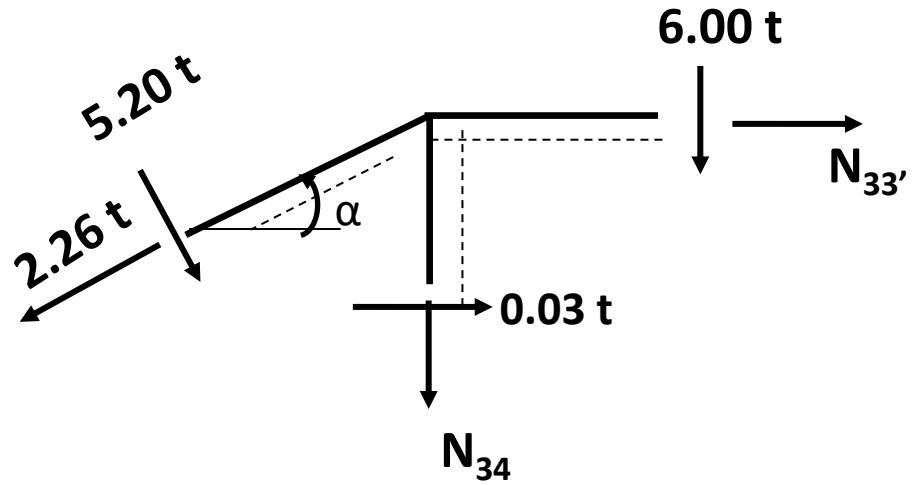
$$-N_{21} \frac{6.0}{6.32} - 5 \frac{6.0}{6.32} - 6.20 + 0.52 * \frac{2}{6.32} = 0 \quad N_{32} = -11.35 t$$

$$N_{23} \frac{6.0}{6.32} + 6.20 * \frac{2}{6.32} - 0.52 = 0 \quad N_{23} = -1.54 t$$



$$N_{32} - N_{23} - 2 * 6 * \frac{2}{6.32} = 0 \quad N_{32} = 2.26 t$$

7. Normal kuvvetlerinin hesabı



$$\sin\alpha = \frac{2}{6.32} \quad \cos\alpha = \frac{6}{6.32}$$

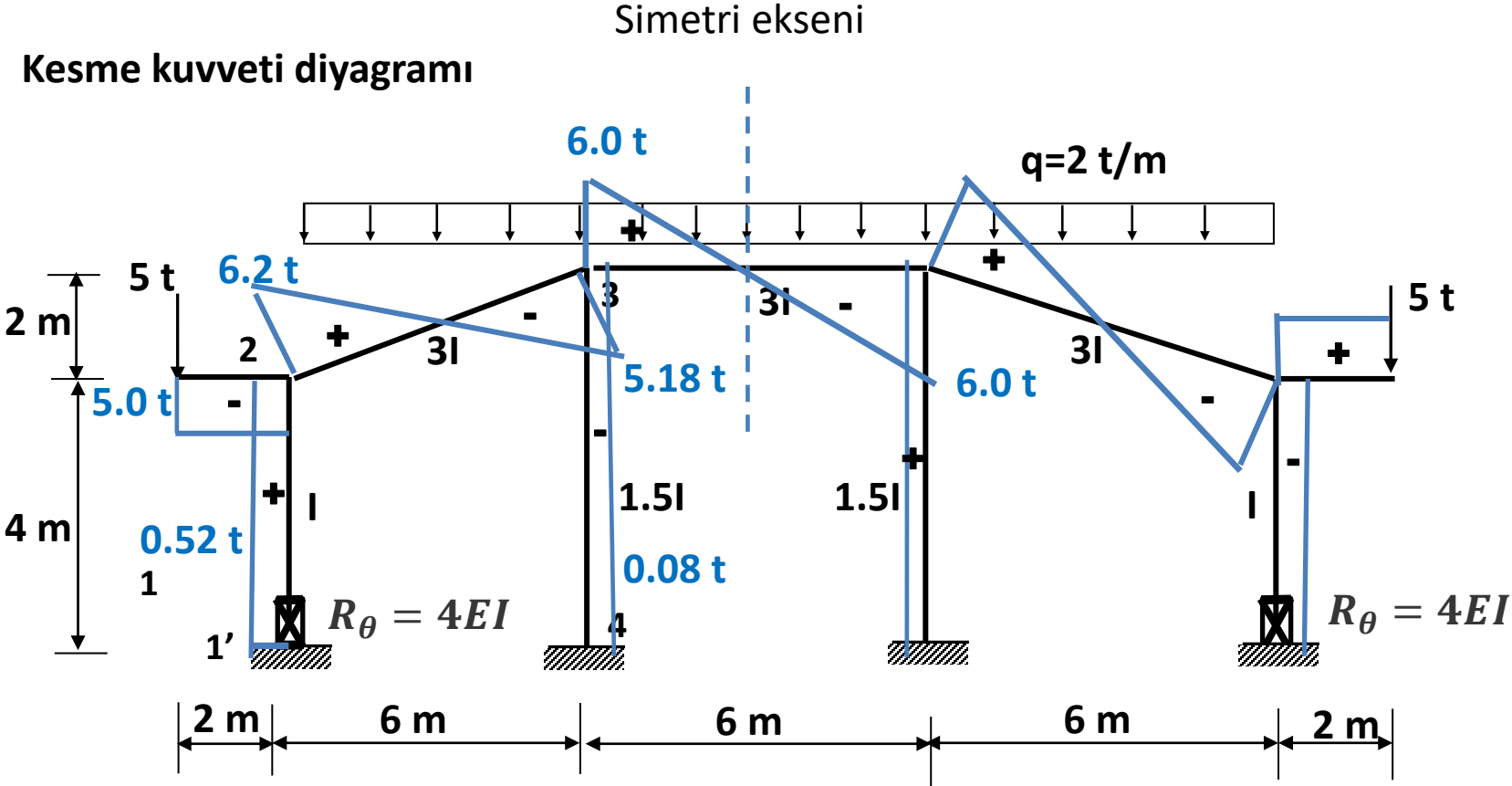
$$N_{33'} = 2.26 * \frac{6.00}{6.32} - 5.20 \frac{2.00}{6.32} - 0.03 = 0.43 t$$

$$N_{33'} = 0.43 t$$

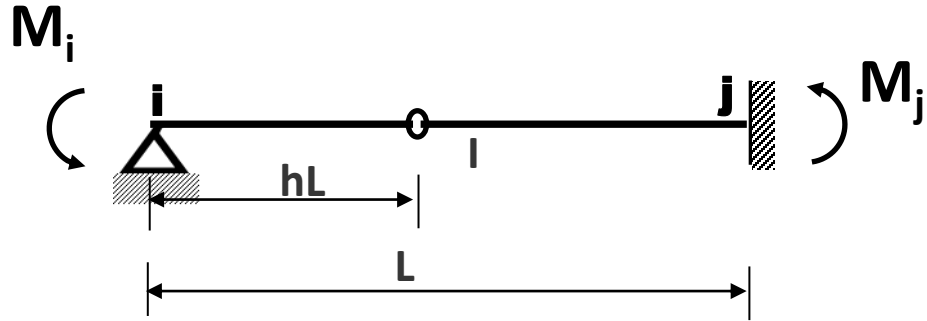
$$N_{34} = -6.00 - 5.20 * \frac{6.00}{6.32} - 2.26 \frac{2.00}{6.32} = -11.56 t$$

$$N_{34} = -11.56 t$$

6. Kesme kuvveti diyagramı



MAFSALLI SİSTEMLER



$$\theta_i = \frac{M_i}{3EI} \frac{3h^2 - 3h + 1}{h^2}$$

$$M_j = M_i \frac{1 - h}{h}$$

$$\frac{M_j}{M_i} = \mu_{ij} = \frac{1 - h}{h}$$

$$h = 1 \text{ için } \mu_{ij} = 0$$

$$h = \frac{2}{3} \text{ için } \mu_{ij} = \frac{1}{2} \quad h = 0.5 \text{ için } \mu_{ij} = 1$$

$$h = 0 \text{ için } \mu_{ij} = \infty$$

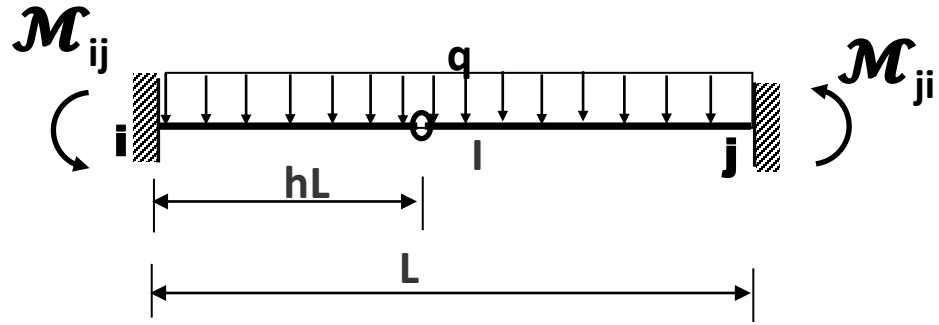
$$\theta_i = 1 \quad M_i = m_{i\theta_i}$$

$$m_{i\theta_i} = \frac{3EI}{L} \frac{h^2}{3h^2 - 3h + 1}$$

$$h = 1 \text{ ise } m_{i\theta_i} = m_{i\theta_i} = \frac{3EI}{L}$$

$$h = \frac{2}{3} \text{ ise } m_{i\theta_i} \text{ maksimum olur.}$$

$$m_{\theta_{i_{\max}}} = \frac{4EI}{L}$$



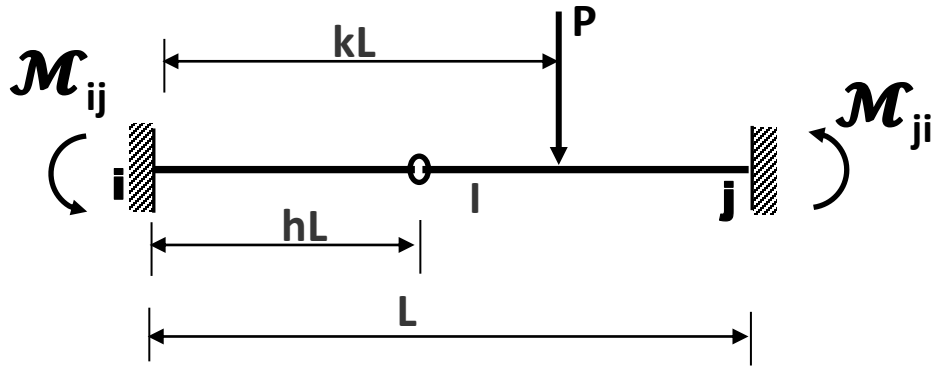
$$\mathcal{M}_{ij} = \frac{qL^2}{8} \frac{h(6h^2 - 8h + 3)}{3h^2 - 3h + 1}$$

$$\mathcal{M}_{ji} = -\frac{qL^2}{8} \frac{(1-h)(6h^2 - 8h + 3)}{3h^2 - 3h + 1}$$

özel hal $h = 0$ $\mathcal{M}_{ij} = 0$ $\mathcal{M}_{ji} = \frac{qL^2}{8}$

$h = 1$ $\mathcal{M}_{ij} = \frac{qL^2}{8}$ $\mathcal{M}_{ji} = 0$

$h = 0.5$ $\mathcal{M}_{ij} = -\mathcal{M}_{ji} = \frac{qL^2}{8}$



$$\mathcal{M}_{ij} = PL \left[k - \frac{k^2 h (3h - k)}{2(3h^2 - 3h + 1)} \right] \quad k \leq h$$

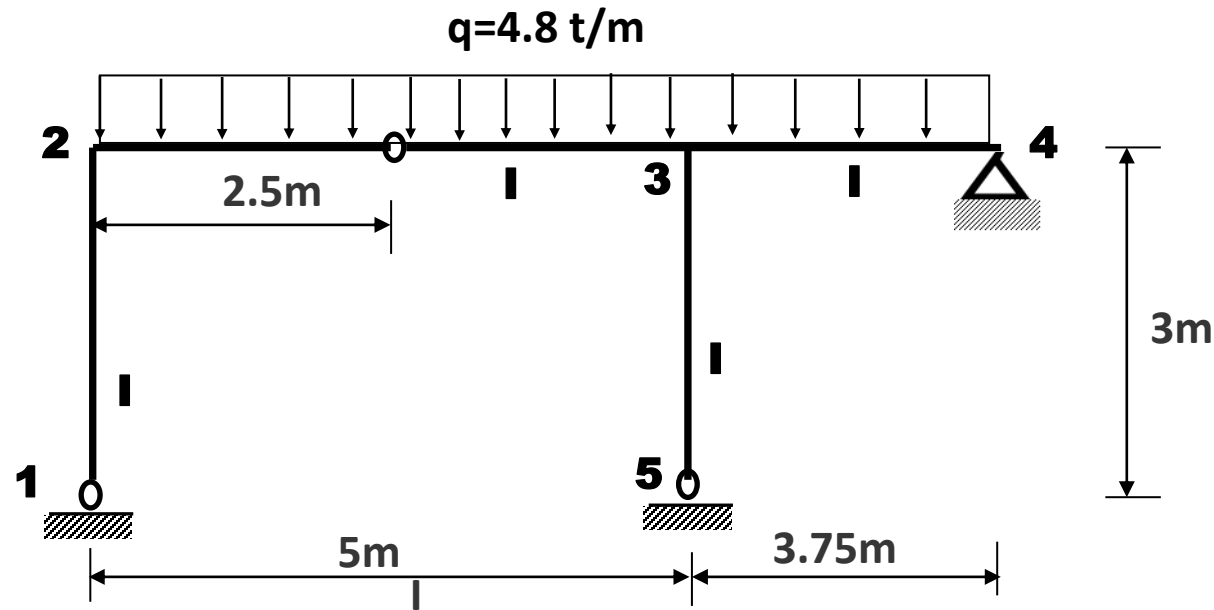
$$\mathcal{M}_{ji} = -PL \left[\frac{k^2 (1 - h)(3h - k)}{2(3h^2 - 3h + 1)} \right] \quad k \leq h$$

$$\mathcal{M}_{ij} = PL \left[\frac{h(1 - k)^2 (k + 2 - 3h)}{2(3h^2 - 3h + 1)} \right] \quad k \geq h$$

$$\mathcal{M}_{ji} = -PL \left[1 - k - \frac{(1 - k)^2 (1 - h)(k + 2 - 3h)}{2(3h^2 - 3h + 1)} \right] \quad k \geq h$$

$$k = h \text{ ise } \quad \mathcal{M}_{ij} = PL \frac{h(1 - h)^3}{3h^2 - 3h + 1} \quad \mathcal{M}_{ji} = -PL \frac{h^3(1 - h)}{3h^2 - 3h + 1}$$

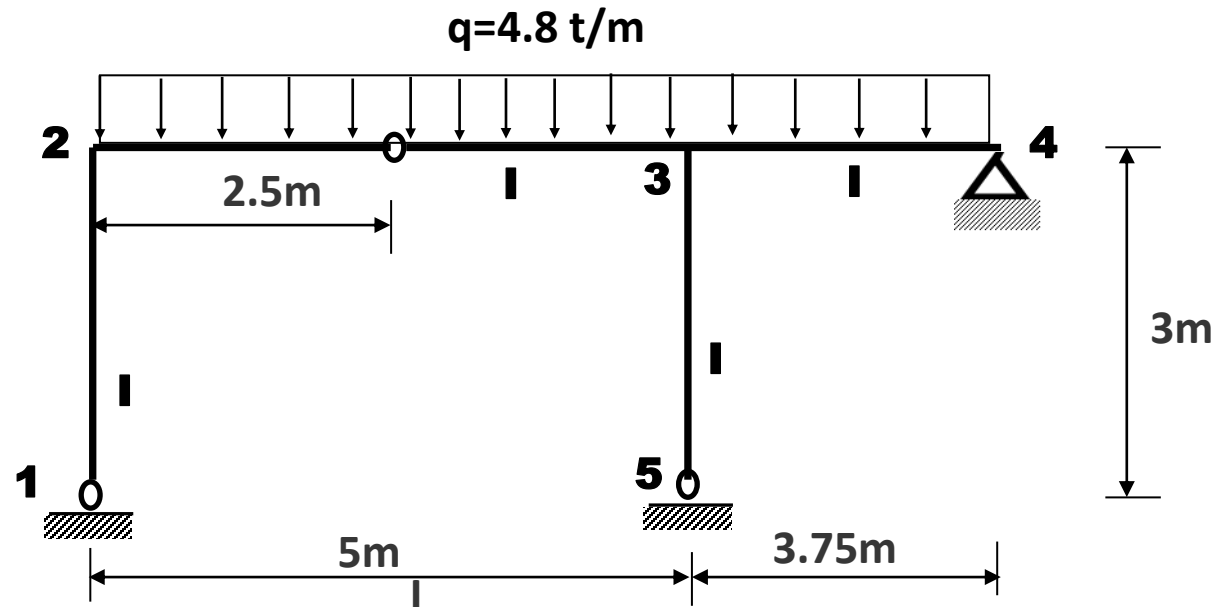
UYGULAMA 4



1. Ankastrelik momentleri:

$$\mathcal{M}_{23} = -\mathcal{M}_{32} = \frac{4.8 * 5^2}{8} = 15 \text{ tm} \quad \mathcal{M}_{34} = \frac{4.8 * 3.75^2}{8} = 8.44 \text{ tm}$$

UYGULAMA



2. Dağıtma sayıları:

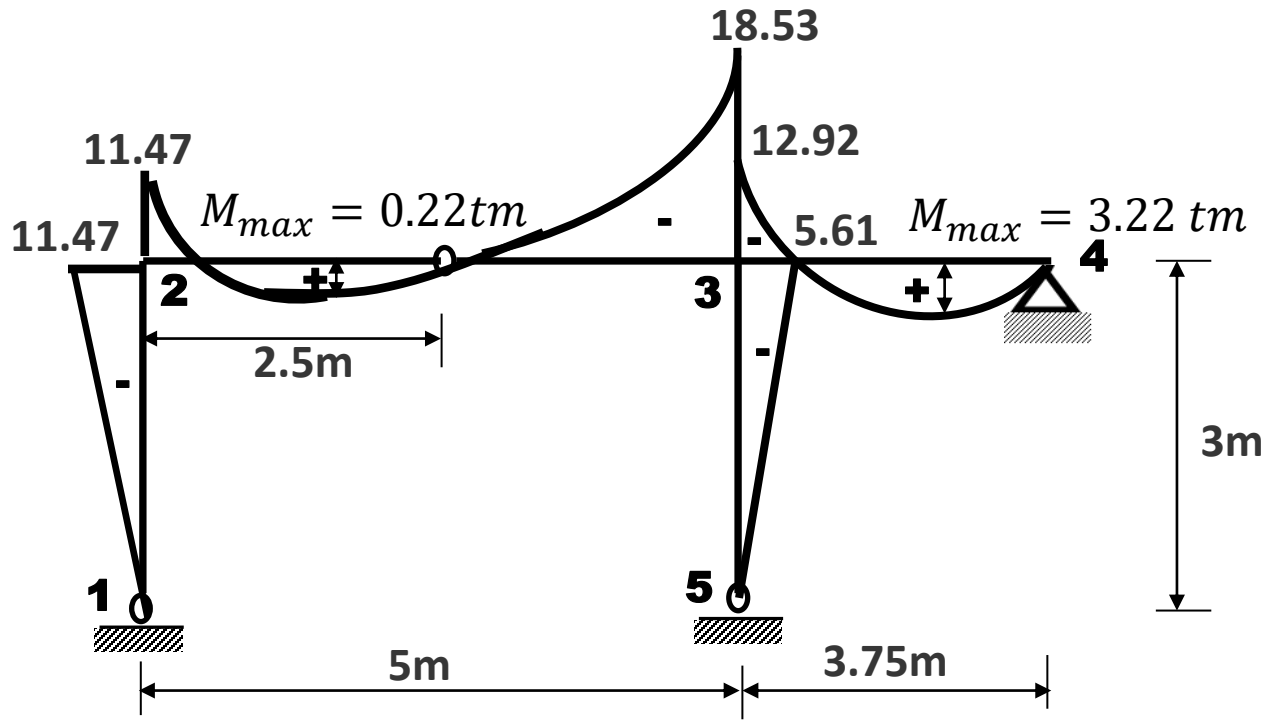
$$r_{21} = \frac{\frac{3EI}{3}}{\frac{3EI}{3} + \frac{3EI}{5}} = 0.625 \quad r_{23} = \frac{\frac{3EI}{5}}{\frac{3EI}{3} + \frac{3EI}{5}} = 0.375$$

$$r_{32} = \frac{\frac{3EI}{5}}{\frac{3EI}{5} + \frac{3EI}{3} + \frac{3EI}{3.75}} = 0.25 \quad r_{35} = \frac{\frac{3EI}{3}}{\frac{3EI}{5} + \frac{3EI}{3} + \frac{3EI}{3.75}} = 0.417 \quad r_{35} = \frac{\frac{3EI}{3.75}}{\frac{3EI}{5} + \frac{3EI}{3} + \frac{3EI}{3.75}} = 0.333$$

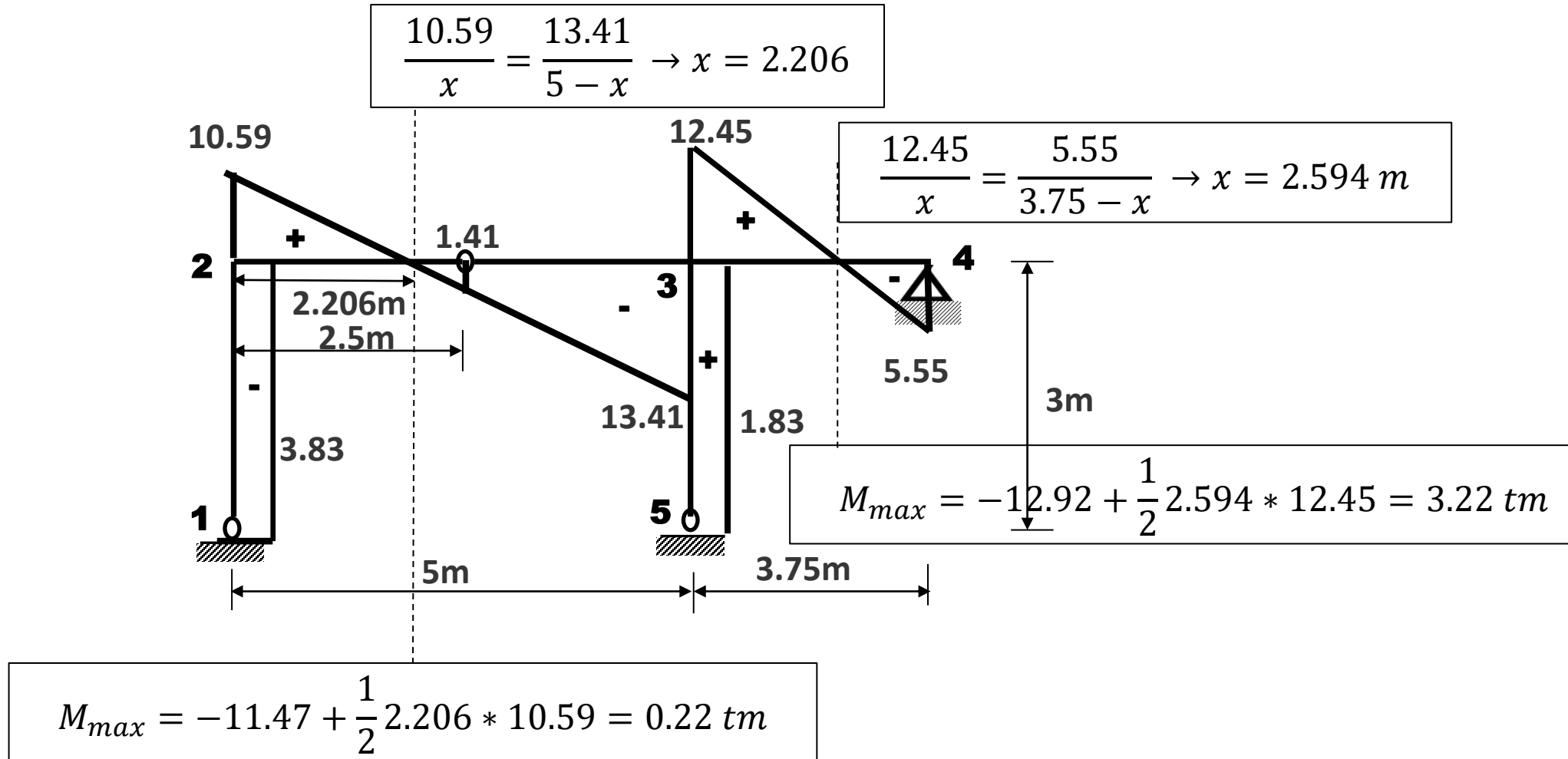
3. Cross dengelemesi:

4. Moment diyagramı:

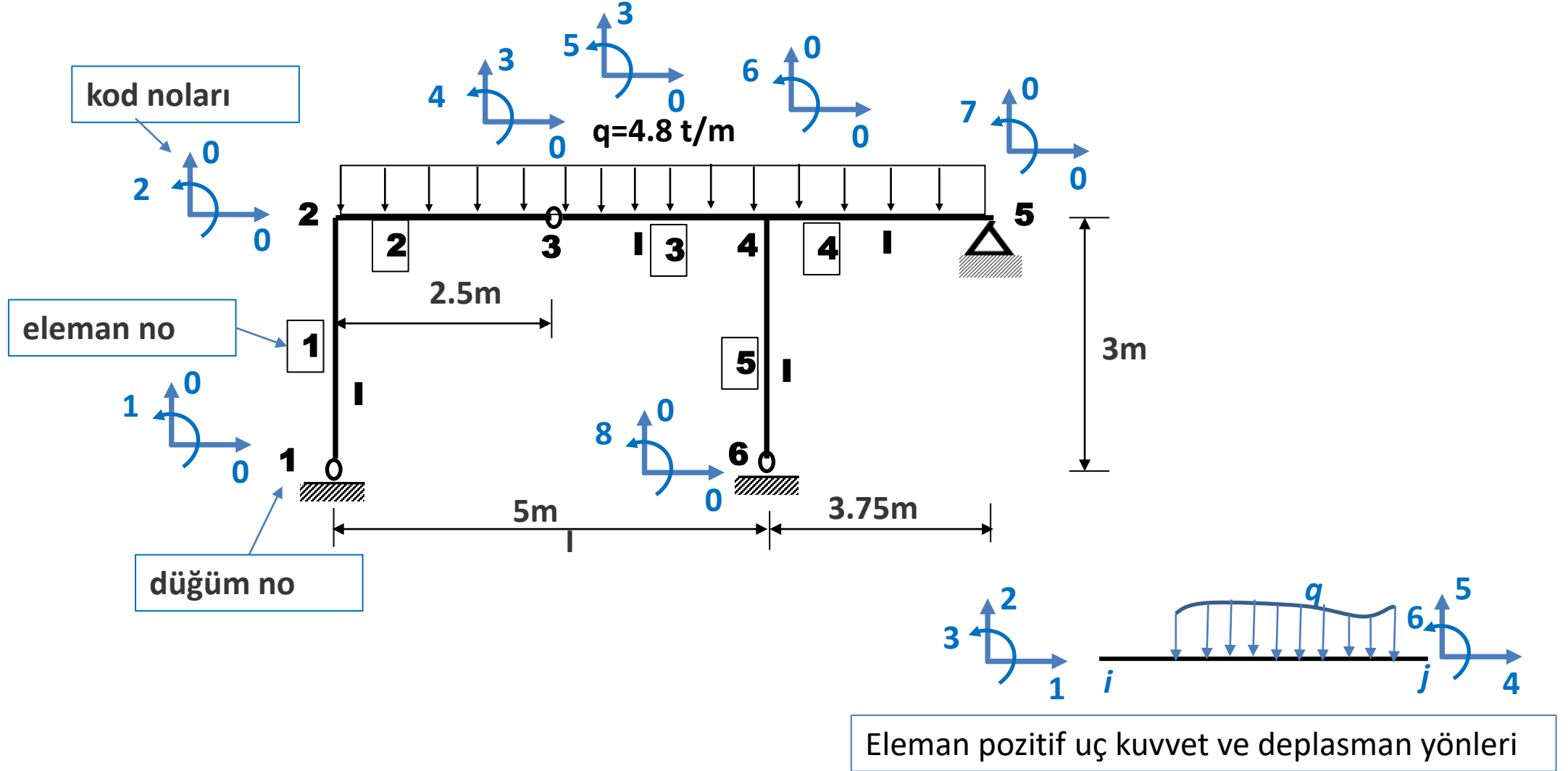
(2-1)	(2-3)		(3-2)	(3-5)	(3-4)
0.625	0.375	$\xrightarrow{1}$ $\xleftarrow{1}$	0.250	0.417	0.333
-9.38	15.00	\rightarrow	-15.00		8.44
-1.90	-5.63	\leftarrow	-5.63		
-0.18	3.04	\rightarrow	3.04	5.08	4.06
-0.11	-1.14	\leftarrow	-1.14		
-0.02	0.29	\rightarrow	0.29	0.48	0.38
-0.01	-0.11	\leftarrow	-0.11		
-0.01	0.03	\rightarrow	0.03	0.05	0.04
-0.01	-0.01	\leftarrow	-0.01		
-11.48	-11.47	\rightarrow	-18.53	5.61	12.92



5. Kesme kuvveti diyagramı:



RİJİTLİK MATRİSİ YÖNTEMİ İLE ÇÖZÜM (SDB88 PROGRAM*)



RIJİTLİK MATRİSİ YÖNTEMİ İLE ÇÖZÜM (SDB88 PROGRAM*)

6] DÜZLEMİ İÇERİSİNDE YUKLU GENEL CERCEVELERİN STATİK HESABI :

MAFSALLI CERCEVE

ELEMAN SAYISI -----= 5
DEPLASMAN SAYISI -----= 8
DUGUM SAYISI -----= 6
ELASTISİTE MODULU -----= 1
YUKLEME SAYISI -----= 1
KAYMA DEFORMASYONLARI İHMAL EDİLİYOR

DUGUM	X	Y
1	0.00	0.00
2	0.00	3.00
3	2.50	3.00
4	5.00	3.00
5	8.75	3.00
6	5.00	0.00

ELEMAN	i	j	BOYU	ALAN	ATALET	K O D	N U M A R A L A R I
1	1	2	3.00	1.000	1.0000	0 0	1 0 0 2
2	2	3	2.50	1.000	1.0000	0 0	2 0 3 4
3	3	4	2.50	1.000	1.0000	0 3	5 0 0 6
4	4	5	3.75	1.000	1.0000	0 0	6 0 0 7
5	4	6	3.00	1.000	1.0000	0 0	6 0 0 8

*Dünder, C., Kırıl, E., Mengi, Y., Yapı Mekaniğinde Bilgisayar Programları, Genişletilmiş 3. Baskı, Teknik Yayınevi, 1987.

YUKLEME NO = 1

MAFSALLI CERCEVE

ANKASTRELİK UC KUVVETLERİ

ELEMAN	Ni	Ti	Mi	Nj	Tj	Mj
1	0.000	0.000	0.000	0.000	0.000	0.000
2	0.000	-6.000	-2.500	0.000	-6.000	2.500
3	0.000	-6.000	-2.500	0.000	-6.000	2.500
4	0.000	-9.000	-5.625	0.000	-9.000	5.625
5	0.000	0.000	0.000	0.000	0.000	0.000

UC KUVVETLERİ

ELEMAN	Mij	Mji	Tij	Tji	Nj	ACIKLIK M.
1	0.00	-11.48	-3.83	3.83	0.00	
2	11.48	-0.00	10.59	1.41	0.00	0.21
3	-0.00	-18.52	-1.41	13.41	0.00	
4	12.92	0.00	12.45	5.55	0.00	3.21
5	5.60	-0.00	1.87	-1.87	0.00	