YAPI STATİĞİ 1

DERS NOTLARI 4

Prof. Dr. Cengiz Dündar

Kesit Tesiri Diyagramları

Kesit zorlarının sistem üzerindeki değişimlerinin ifade eden grafiksel çizimlere **kesit tesiri diyagramları** denilmektedir.

Düzlem çubuk sistemlerde **M, N, T** diyagramları olmak üzere 3 çeşit kesit tesiri diyagramı bulunmaktadır. Söz konusu diyagramlar sistem üzerinde yatay düzlemde gösterilirler ve ilgili noktalarda ki ordinat değerleri sistemin o noktasına ait kesit tesiri değerini belirtmektedir.

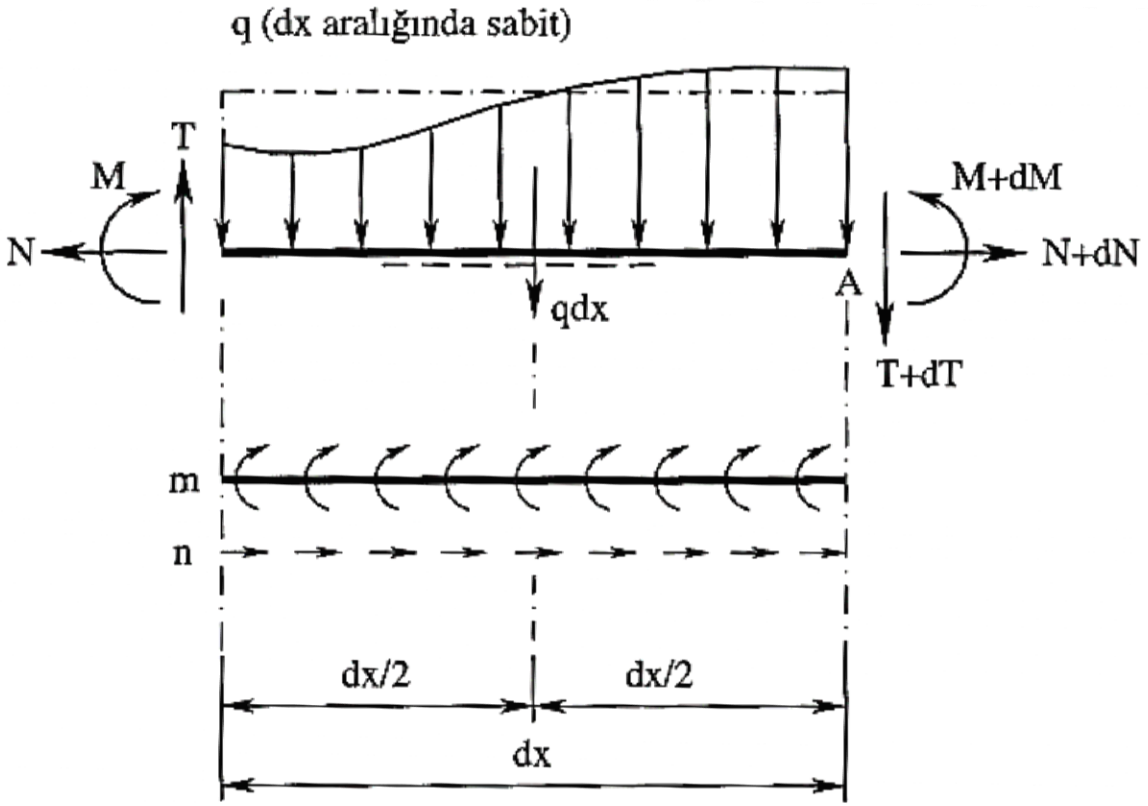
Kesit Tesiri Diyagramlarının Çiziminde İzlenen Yol

1) Kesit zorları sistem üzerinde gösterilir ve çubuk eksenine dik olarak çizilirler.
2) Kesit tesirleri ölçekli olarak çizilmeli ve ordinatları doğrultusunda taranmalıdırlar.

3) Diyagramların üzerine başlıca noktalardaki değerleri yazılmalı ve bölgenin işareti konulmalıdır. (çekme veya basınç olacak şekilde)
4) N ve T diyagramlarında pozitif değerler bakış yönünün tersi tarafında, M diyagramında ise bakış yönü ile aynı tarafta olmalıdır. Böylelikle M diyagramı daima kesitin uzama liflerinin olduğu tarafta çizilmiş olacaktır.
5) Verilen sistem yeterli sayıda bölgeye ayrılır, kesimler yapılır ve her bölge için seçilen değişkenin fonksiyonu olarak (x: mesafe değişkeni), N(x), T(x) ve M(x) değerleri hesaplanır ve fonksiyonları çizilir.
6) Diyagramların çiziminde kritik bazı kesitler vardır. Bu durumda sistem üzerinde sınırlı sayıda kesitte kesimler yapılarak kesit zorları belirlenir ve M, N, T diyagramları çizilir. Bu çözüm için en kolay yoldur. Bu nedenle kritik kesitlerin bilinmesi önemlidir.
7) Kolay yoldan bir sistemin M, N, T diyagramlarının çizilebilmesi için bazı bilgiler gereklidir.

Yük-Kesme Kuvveti ve Eğilme Momenti Arasındaki Bağıntılar

(q, T, M Arasındaki Bağıntılar)



q yayılı yükü dx aralığında sabit olmak üzere, izdüşüm ve moment denge denklemleri yazılır.

$$(\rightarrow +)\ \Sigma F_x = 0 \Rightarrow -N + ndx + N + dN = 0 \Rightarrow \frac{dN}{dx} = -n$$

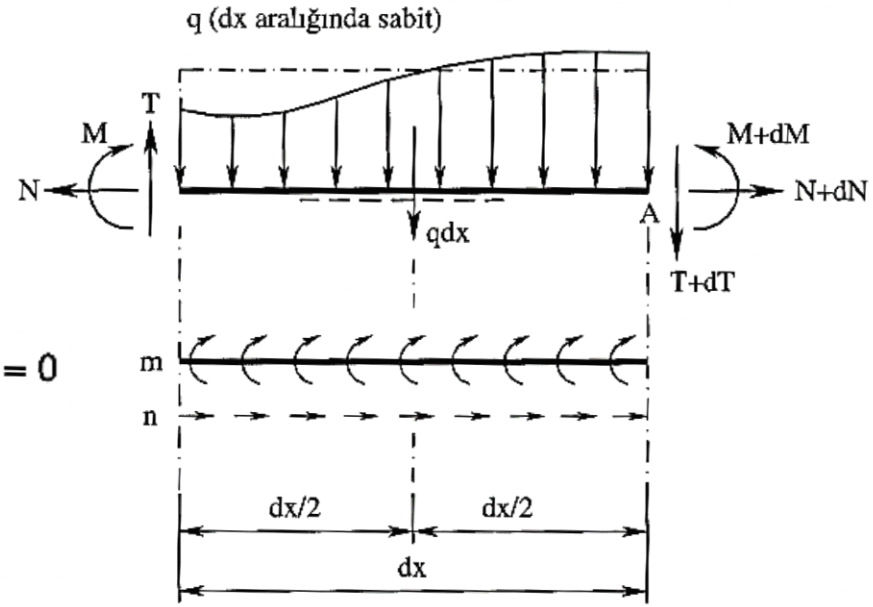
$$n = 0 \Rightarrow \frac{dN}{dx} = 0 \Rightarrow N \rightarrow \text{sabit}$$

$$(\uparrow +)\ \Sigma F_y = 0 \Rightarrow T - qdx - T - dT = 0 \Rightarrow \frac{dT}{dx} = -q$$

$$(\circlearrowleft +)\ \Sigma M_A = 0 \Rightarrow M + Tdx + mdx - qdx\frac{dx}{2} - M - dM = 0$$

$$qdx\frac{dx}{2} \cong 0 \Rightarrow \frac{dM}{dx} = T + m$$

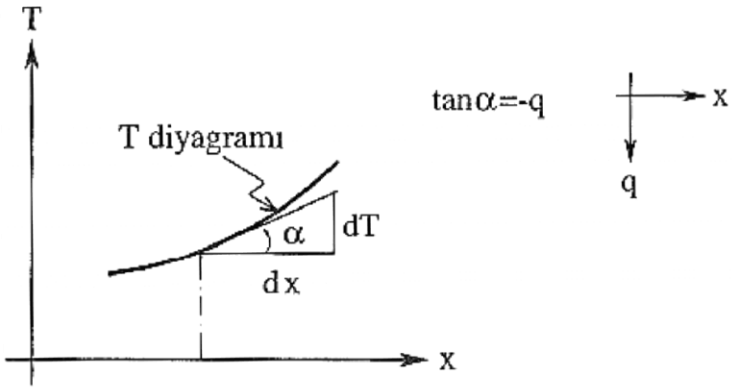
$$m = 0 \Rightarrow \frac{dM}{dx} = T$$



$\frac{dT}{dx} = -q$

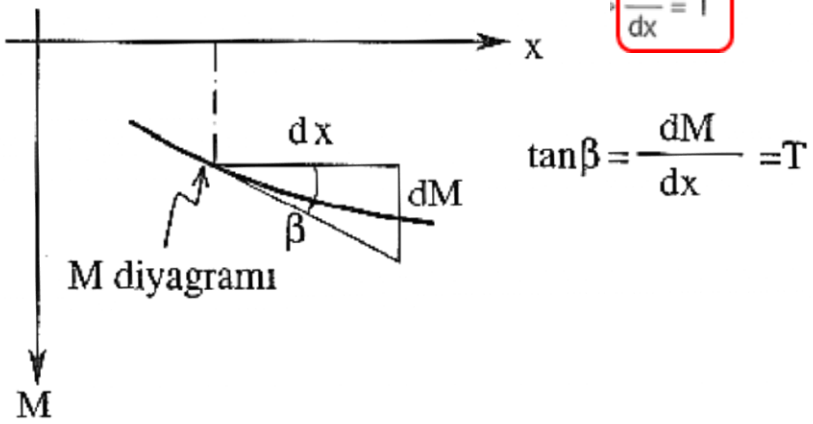
Sonuçlara göre;

1- Kesme kuvveti fonksiyonunun türevi (-) işaretle yayılı yükün şiddetine eşittir. Yani, kesme kuvveti diyagramının herhangi bir noktasındaki eğimi (-) işaretle o noktadaki yayılı yükün şiddetine eşittir.



2- Eğilme momenti fonksiyonunun türevi (yayılı moment $m = 0$ olması halinde) kesme kuvvetine eşittir. Yani, eğilme momenti diyagramının herhangi bir noktasındaki eğimi o noktadaki kesme kuvvetinin şiddetine eşittir.

$$\frac{dM}{dx} = T$$



$$\tan\beta = \frac{dM}{dx} = T$$

3- Örneğin,

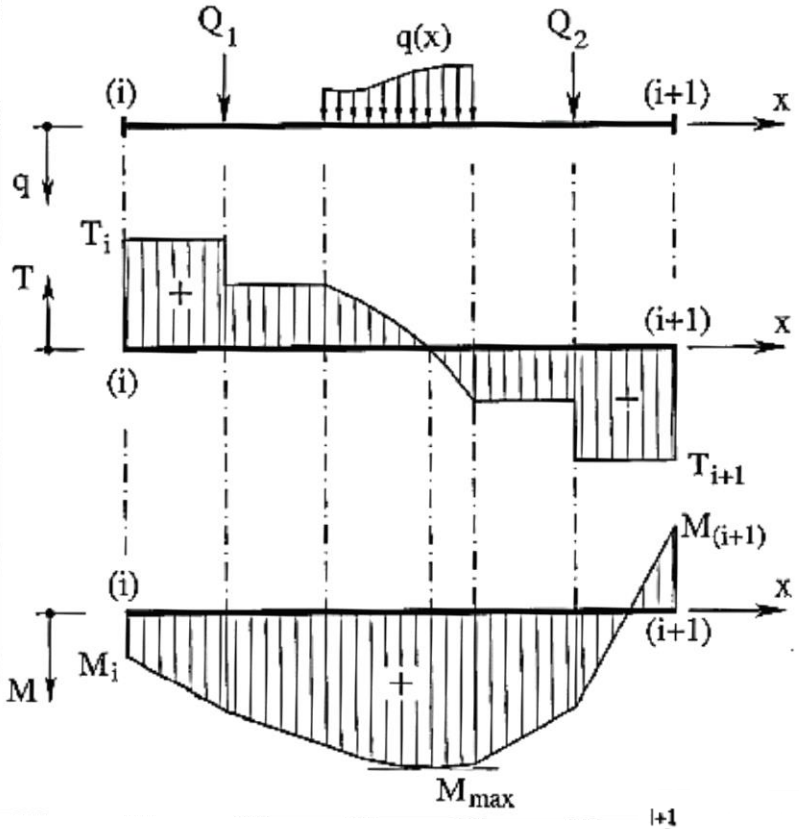
q=0 (yük yok) $\Rightarrow$ T $\rightarrow$ sabit , M $\rightarrow$ 1.º (doğrusal)

q=sabit (düzgün yayılı yük) $\Rightarrow$ T $\rightarrow$ 1.º (doğrusal) , M $\rightarrow$ 2.º (parabol)

q=1.º (doğrusal) (üçgen veya trapez yayılı yük)
$\Rightarrow$ T $\rightarrow$ 2.º (parabol) , M $\rightarrow$ 3.º (parabol)

4- a) Ardışık iki noktadaki kesme kuvvetlerinin farkı, (-) işaretle bu iki nokta arasındaki yüklerin toplamına eşittir.

b) Ardışık iki noktadaki eğilme momentlerinin farkı, bu iki nokta arasındaki kesme kuvveti diyagramının alanına eşittir.



$$\frac{dT}{dx} = -q \Rightarrow dT = -qdx$$

$$\int_i^{i+1} dT = -\left(\int_i^{i+1} q(x)dx + \sum Q_i\right) \Rightarrow T_{i+1} - T_i = -\left(\int_i^{i+1} q(x)dx + \sum Q_i\right)$$

$\left(\int_i^{i+1} q(x)dx + \sum Q_i\right)$: yüklerin toplamı

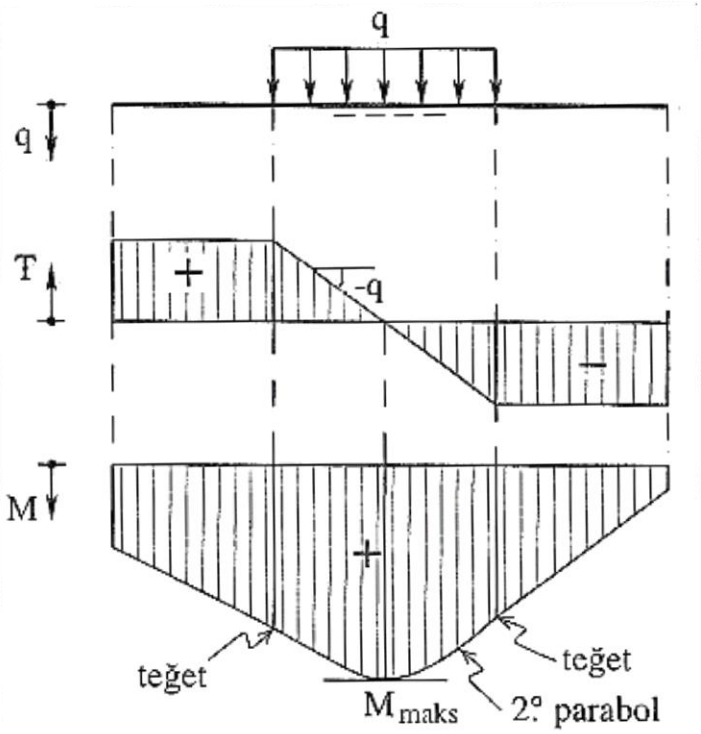
$$\frac{dM}{dx} = T \Rightarrow dM = Tdx$$

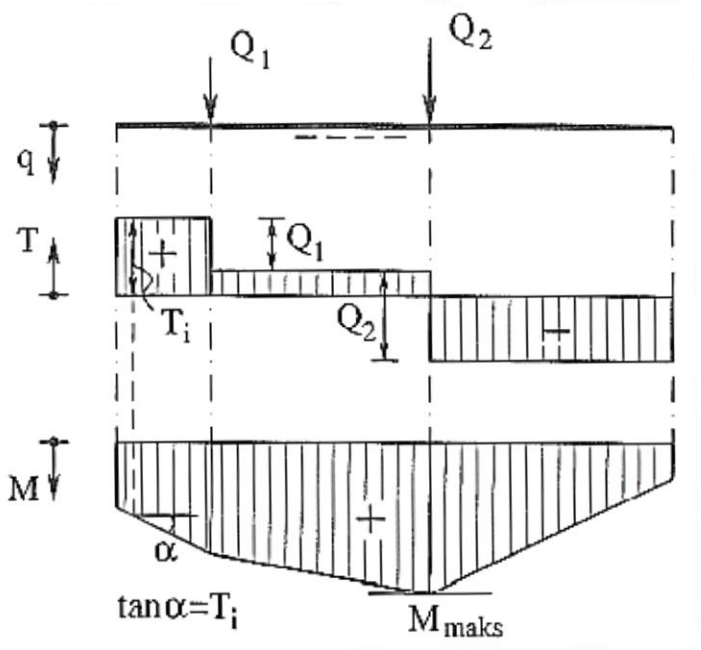
$$\int_i^{i+1} dM = \int_i^{i+1} Tdx \Rightarrow M_{i+1} - M_i = \int_i^{i+1} Tdx$$

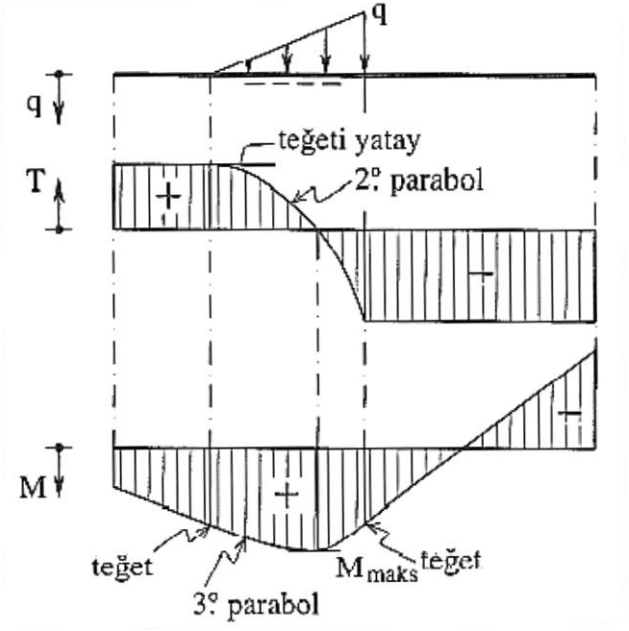
$\int_i^{i+1} Tdx$: kesme kuvveti diyagramının i ile i+1 kesitleri arasındaki alanı

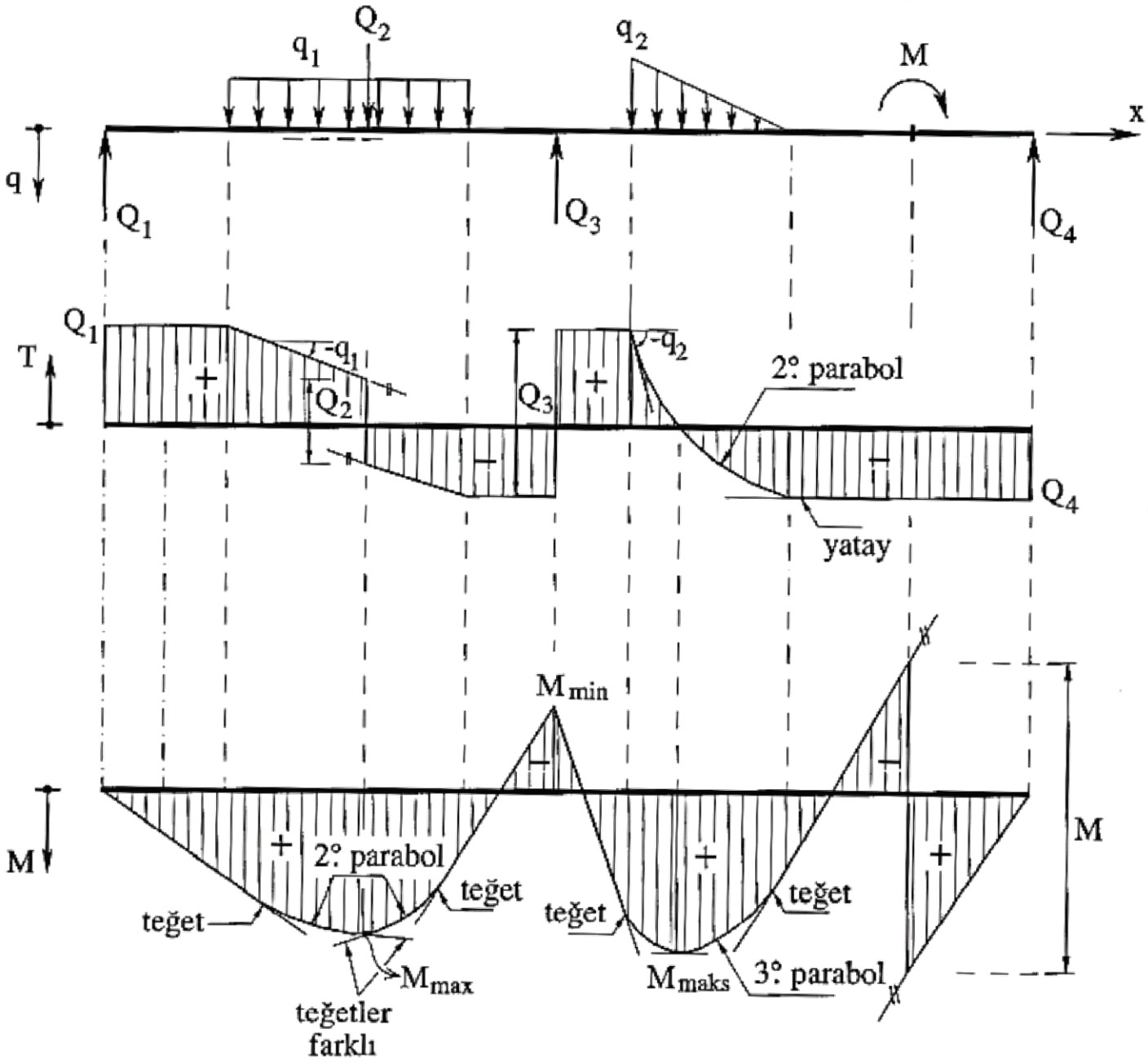
5- Kesme kuvveti diyagramının sıfır olduğu noktalarda eğilme momenti diyagramı ekstremumdan (maksimum veya minimumdan) geçer.

Not : q, T, M arasındaki bu bağıntılar ve sonuçlar, x ekseninin pozitif yönünün sağa doğru olması halinde geçerlidir.


q
q
T
-q
M
teğet
teğet
M_{maks}
2. parabol

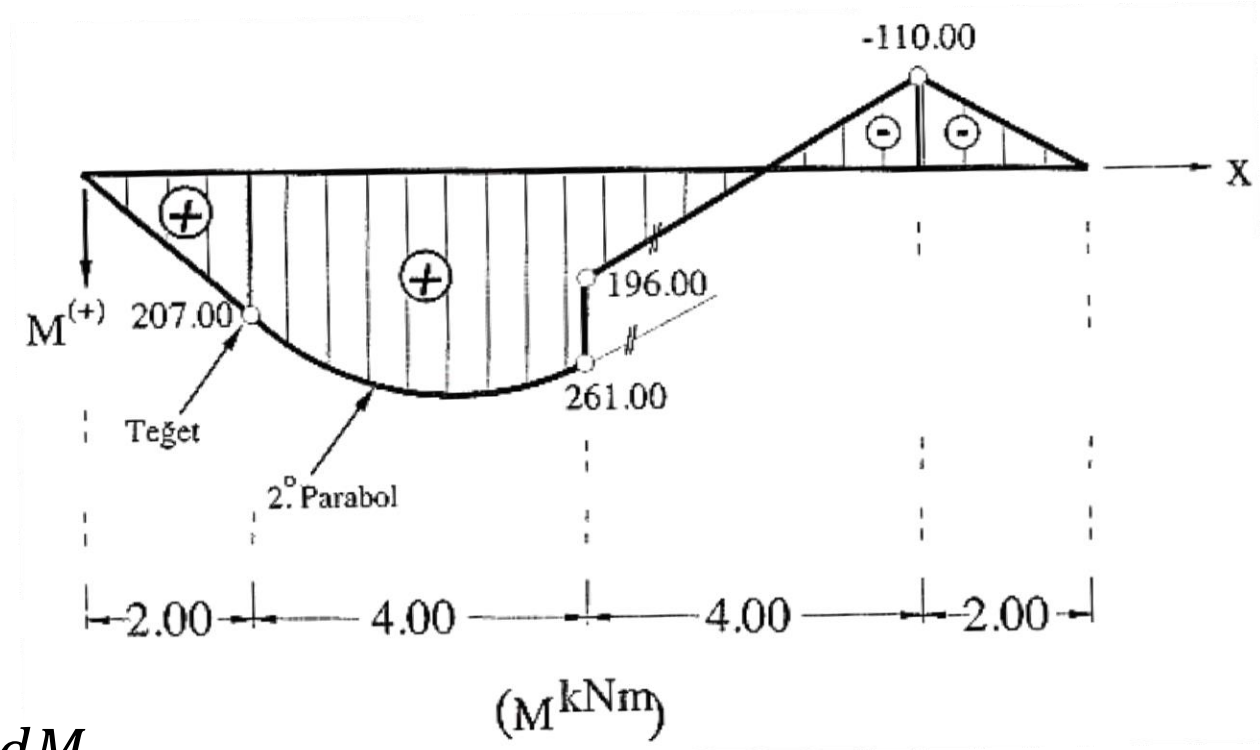

Q_1
Q_2
q
T
Q_1
T_i
Q_2
M
α
$\tan\alpha=T_i$
M_{maks}


q
q
teğeti yatay
T
2. parabol
M
teğet
M_{maks}
teğet
3. parabol


q1
Q2
q2
M
x
q
Q1
Q3
Q4
T
-q1
-q2
2° parabol
yatay
Mmin
M
teğet
Mmax
teğetler farklı
Mmaks
3° parabol

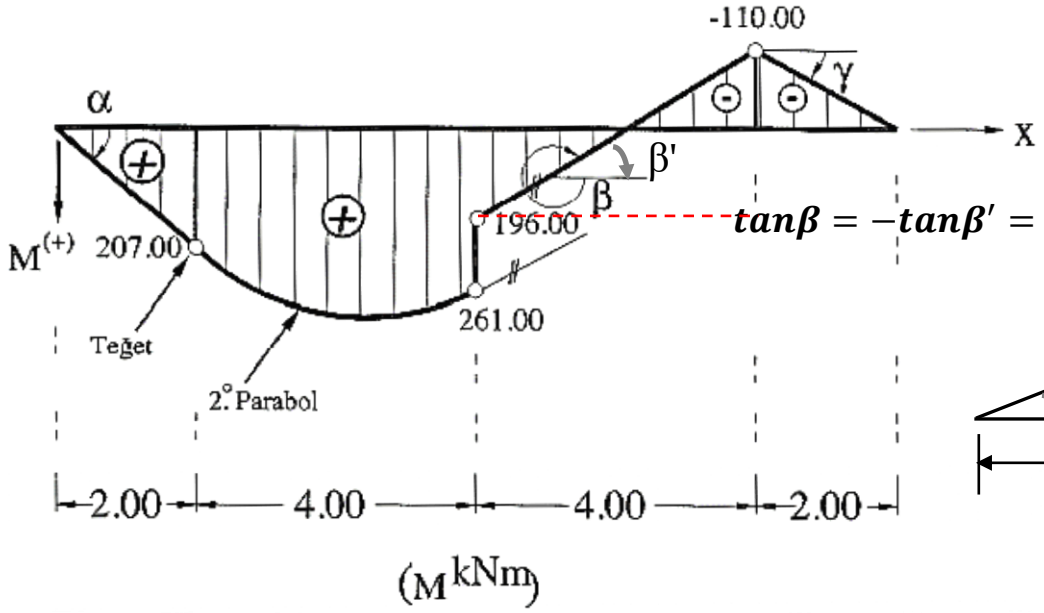
M diyagramı bilinen bir kesitin T ve q (yük) dağılımının elde edilmesi

ÖRNEK

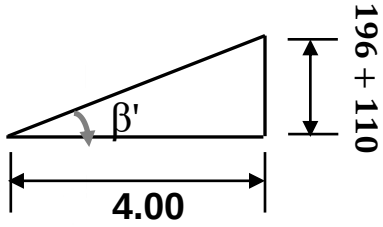


$$\frac{dM}{dx} = T$$

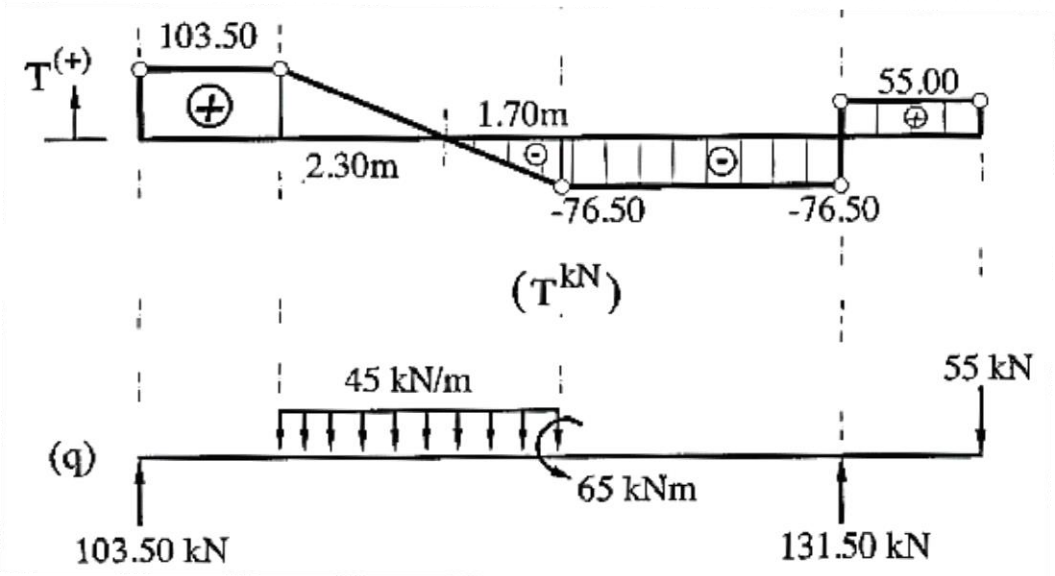
Buna göre eğilme momenti diyagramının üzerinde bulunan doğru parçalarının eğimi kesme kuvveti değerlerini verecektir. Bu doğruların yatayla yaptıkları açılar α, β, γ olup bu açıların tan değerlerinden kesme kuvvetlerinin sayısal değerleri elde edilebilir.



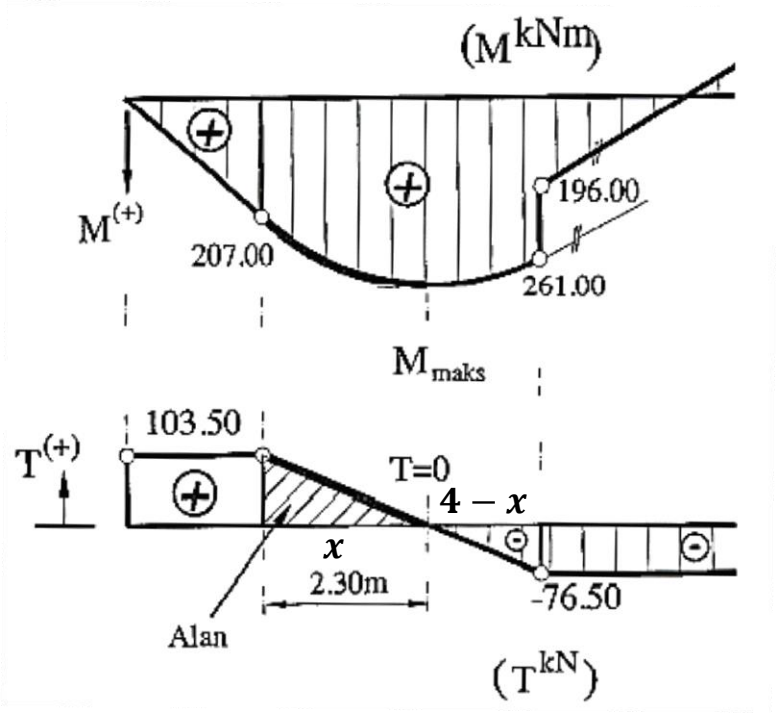
$$tan\beta = -tan\beta' = -\frac{196+110}{4} = -76.5$$



$$\tan\alpha = \frac{207}{2} = 103.5 \quad , \quad \tan\beta = -\frac{(196+110)}{4} = -76.50 \quad , \quad \tan\gamma = \frac{110}{2} = 55$$



Burada maksimum eğilme momenti; kesme kuvveti diyagramının alanından



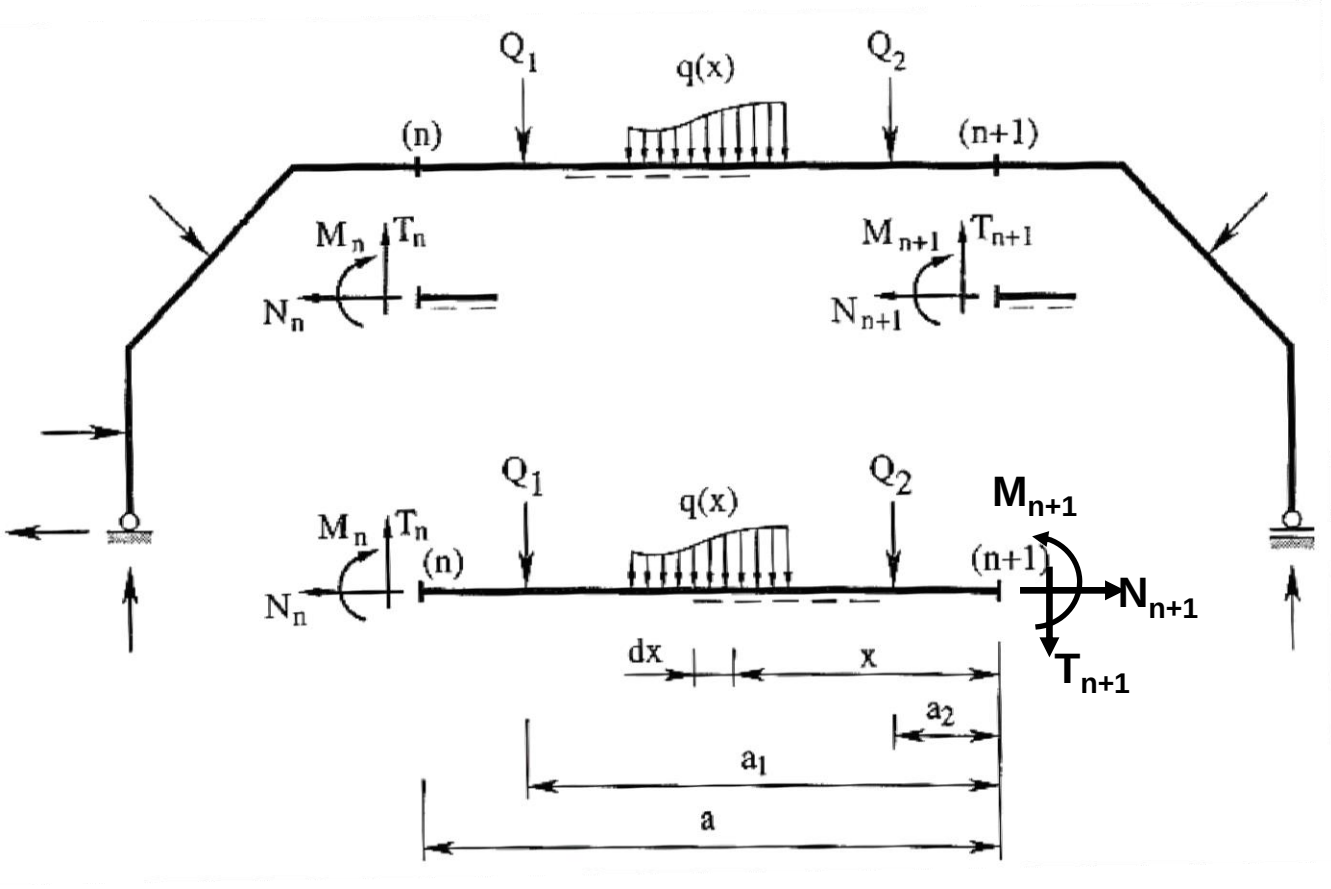
$$\frac{103.5}{x} = \frac{76.5}{4-x} \rightarrow x = 2.30m$$

$$M_{maks} = 207 + \frac{103.5 \times 2.30}{2} = 326.03\,kNm$$ şeklinde hesaplanabilir.

Diyagramların daha kolay yoldan çizilebilmesi için yardımcı notlar;

Not 1:

Sistemin bir (n) kesitindeki kesit zorları belli iken bunun sağındaki (n+1) kesitindeki kesit zorlarını hesaplamak için, **(n)** kesitinin solundaki tüm kuvvetler yerine (n) kesitindeki kesit zorları ve (n ~ n+1) arasındaki dış yükler gözönüne alınabilir.



M_n, T_n, N_n kesit zorları (n) kesitinin solundaki tüm kuvvetlere eşdeğer olduğundan (n+1) kesitindeki kesit zorları, bunlara ve iki kesit arasındaki yüklere bağlı olarak hesaplanabilirler.

$$N_{n+1} = N_n$$

$$T_{n+1} = T_n - \sum Q_i - \int q(x)dx$$

$$M_{n+1} = M_n + T_n a - \sum Q_i a_i - \int q(x)xdx$$

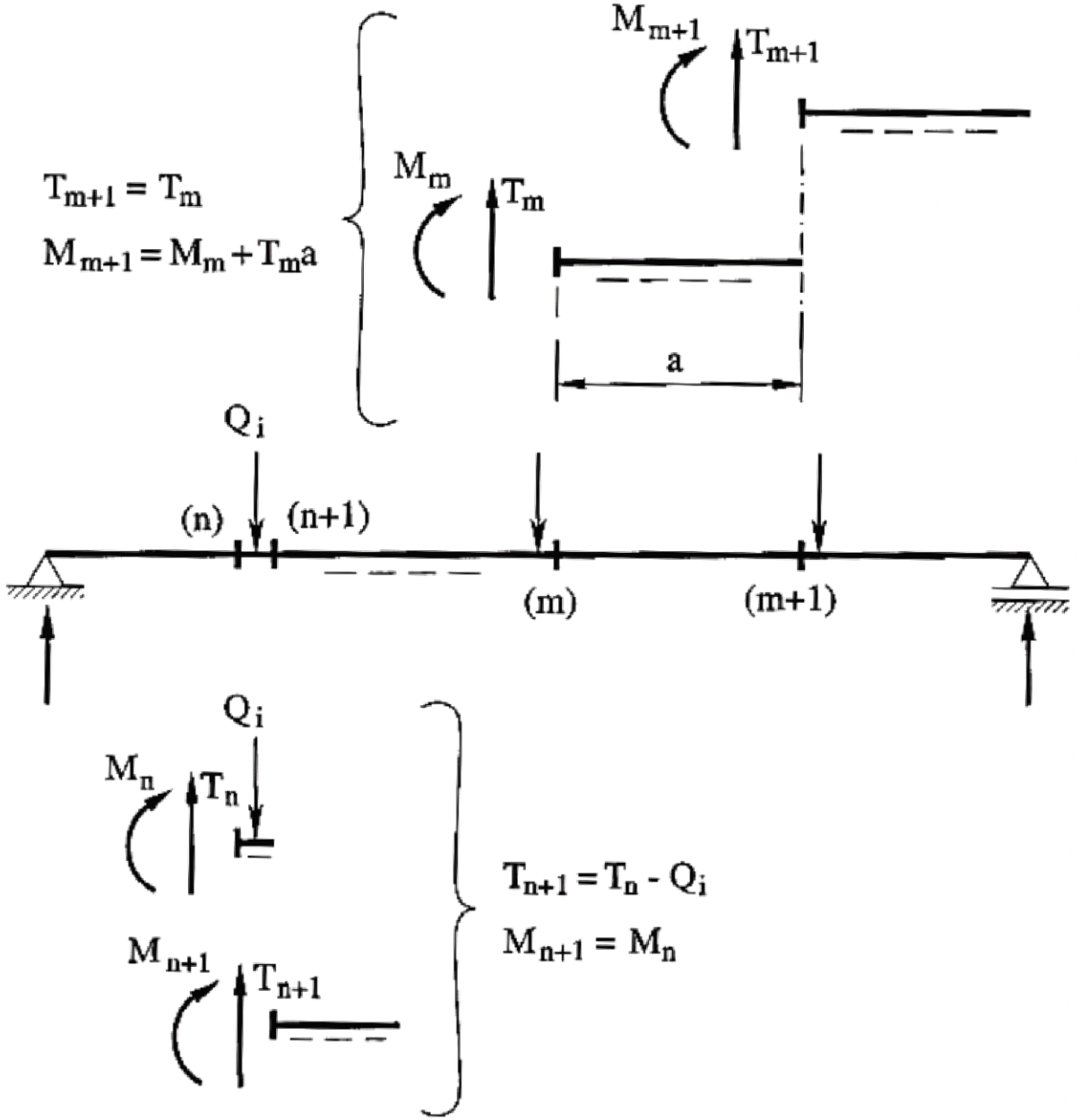
Özel hal : $q(x) = 0$ ise

$$T_{n+1} = T_n - \sum Q_i$$

$$M_{n+1} = M_n + T_n a - \sum Q_i a_i$$

Not : Bu bağıntılar (n)-(n+1) kesitleri arasındaki çubuk ekseninin doğru parçası olması halinde geçerlidir.

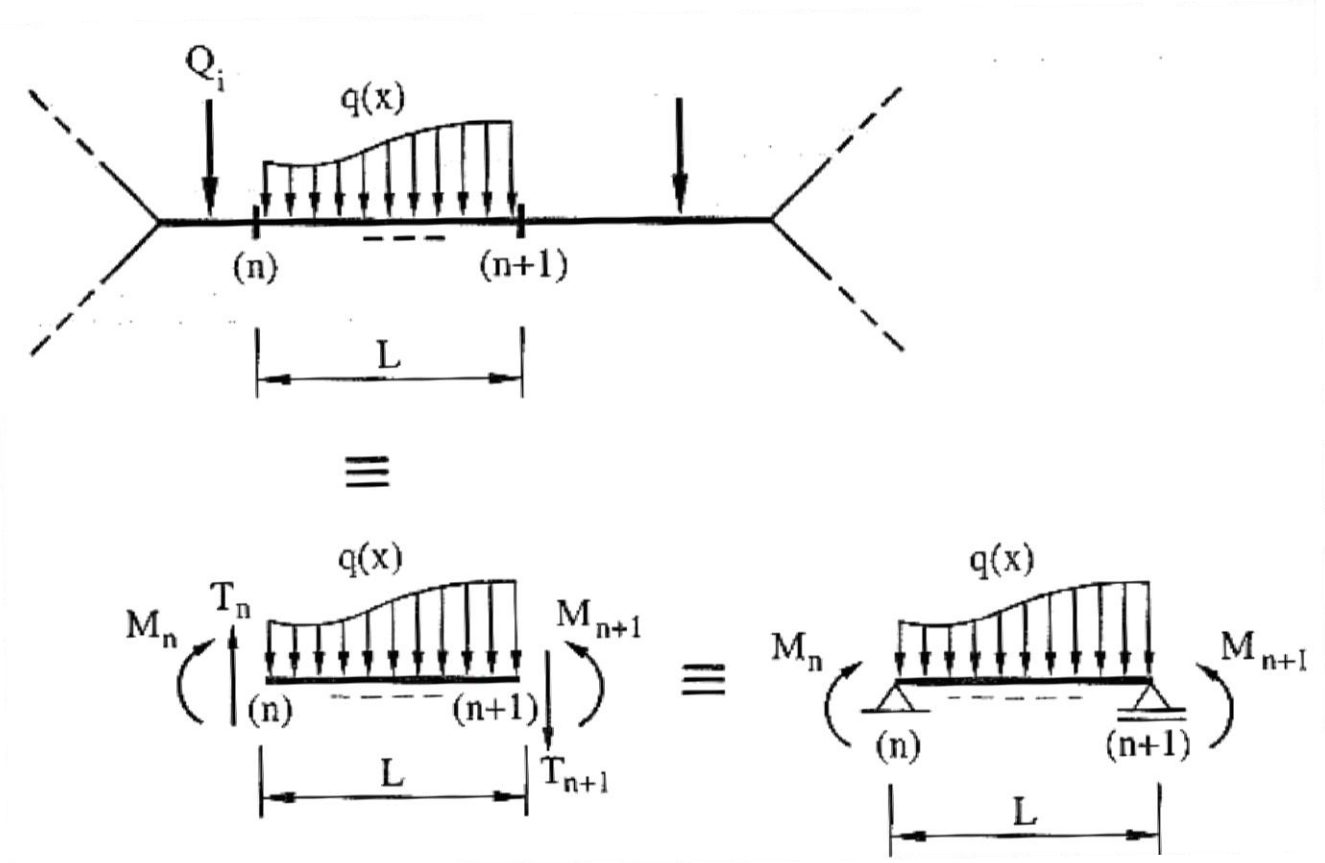
Uygulama

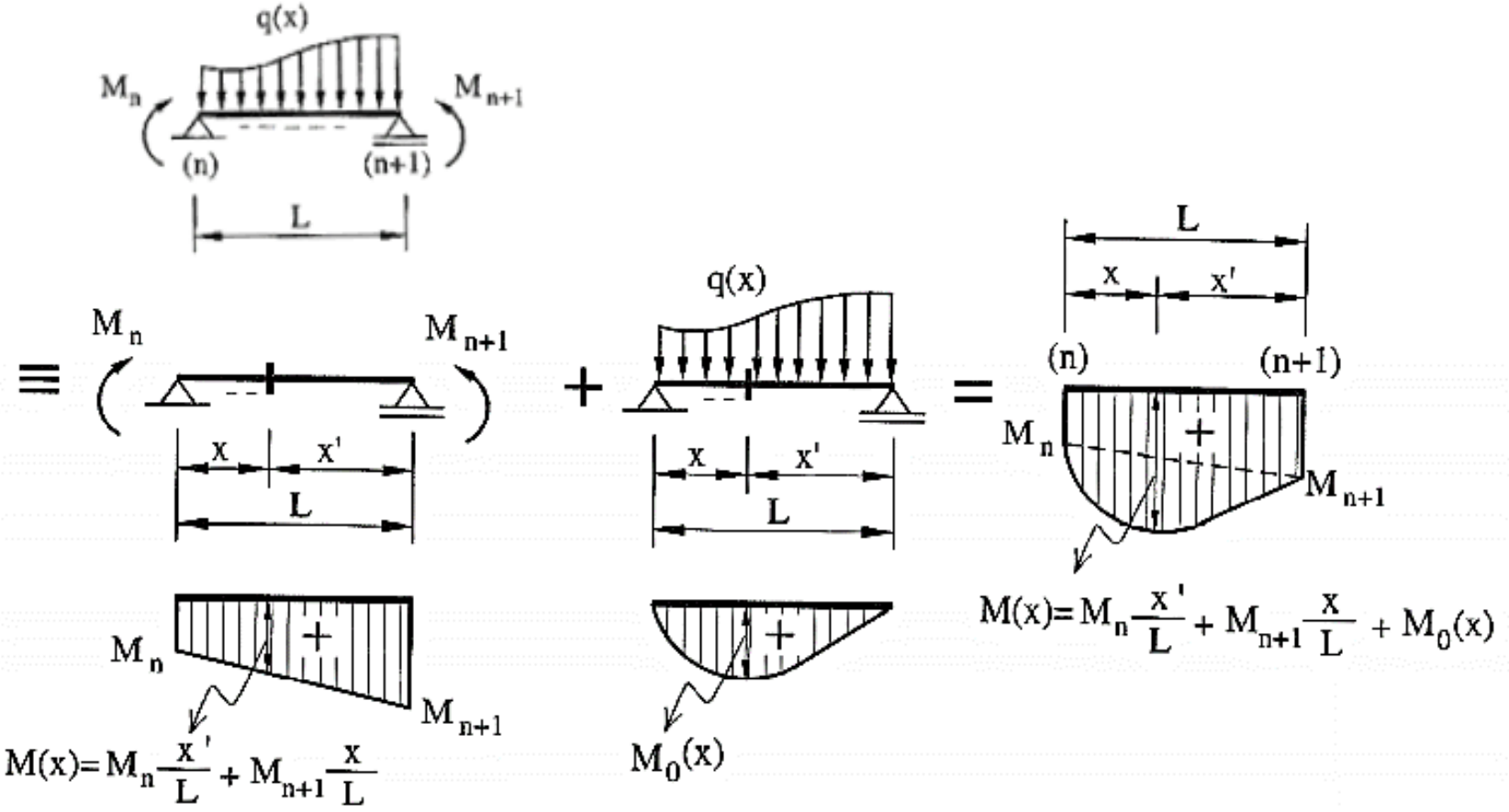

M_{m+1}
T_{m+1}
$T_{m+1} = T_m$
$M_{m+1} = M_m + T_m a$
M_m
T_m
a
Q_i
(n)
(n+1)
(m)
(m+1)
Q_i
M_n
T_n
$T_{n+1} = T_n - Q_i$
$M_{n+1} = M_n$
M_{n+1}
T_{n+1}

Not 2:

Bu notta komşu iki kesitteki eğilme momentleri belirli iken bu iki kesit arasındaki eğilme momenti diyagramının nasıl elde edileceği açıklanacaktır.

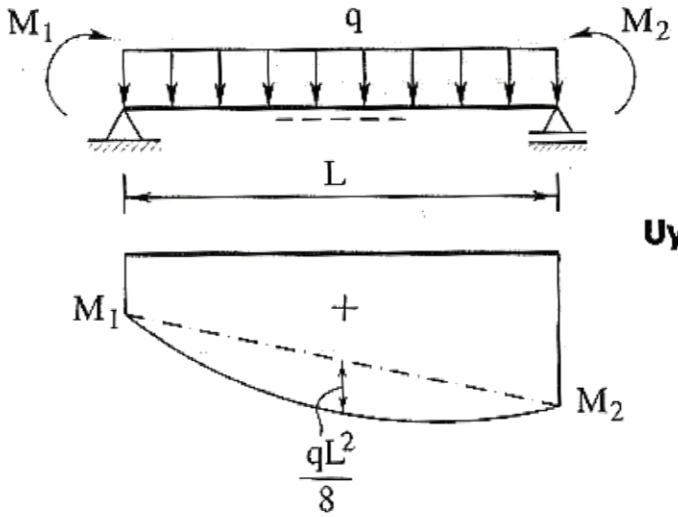
Not : Komşu iki kesit arasındaki çubuk parçasının doğru eksenli olduğu varsayılır.



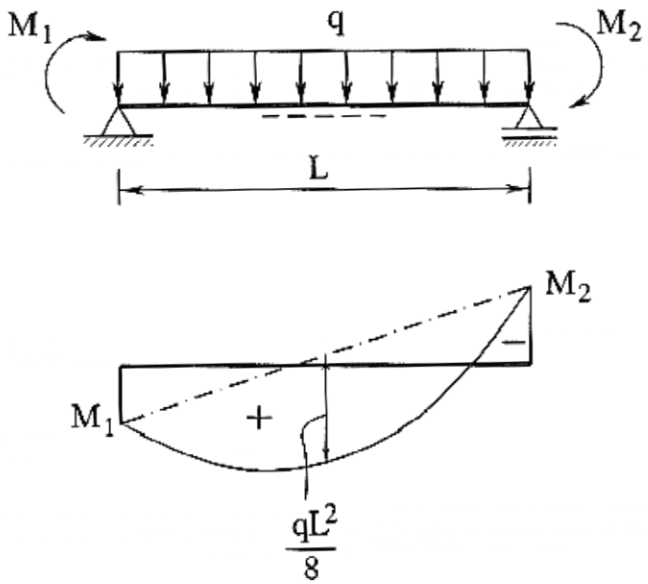


Not : Sistem dengede olduğundan, basit kirişte denge denklemlerini sağlayan mesnet tepkileri T_n ve T_{n+1} e eşittir.

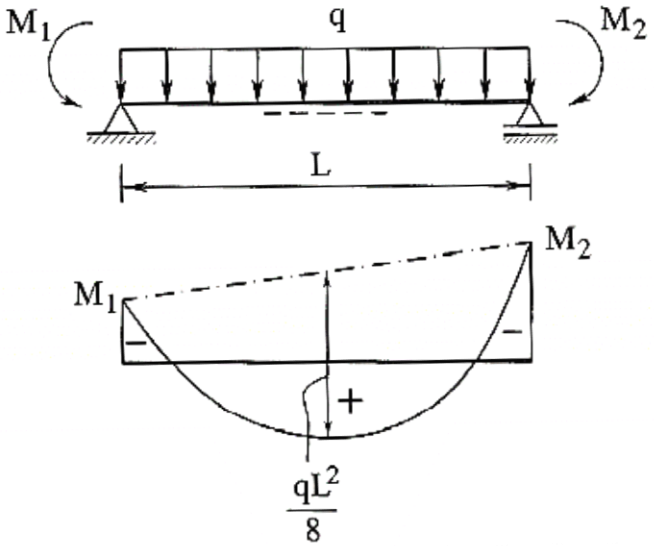
Uygulama:1)



Uygulama:2)

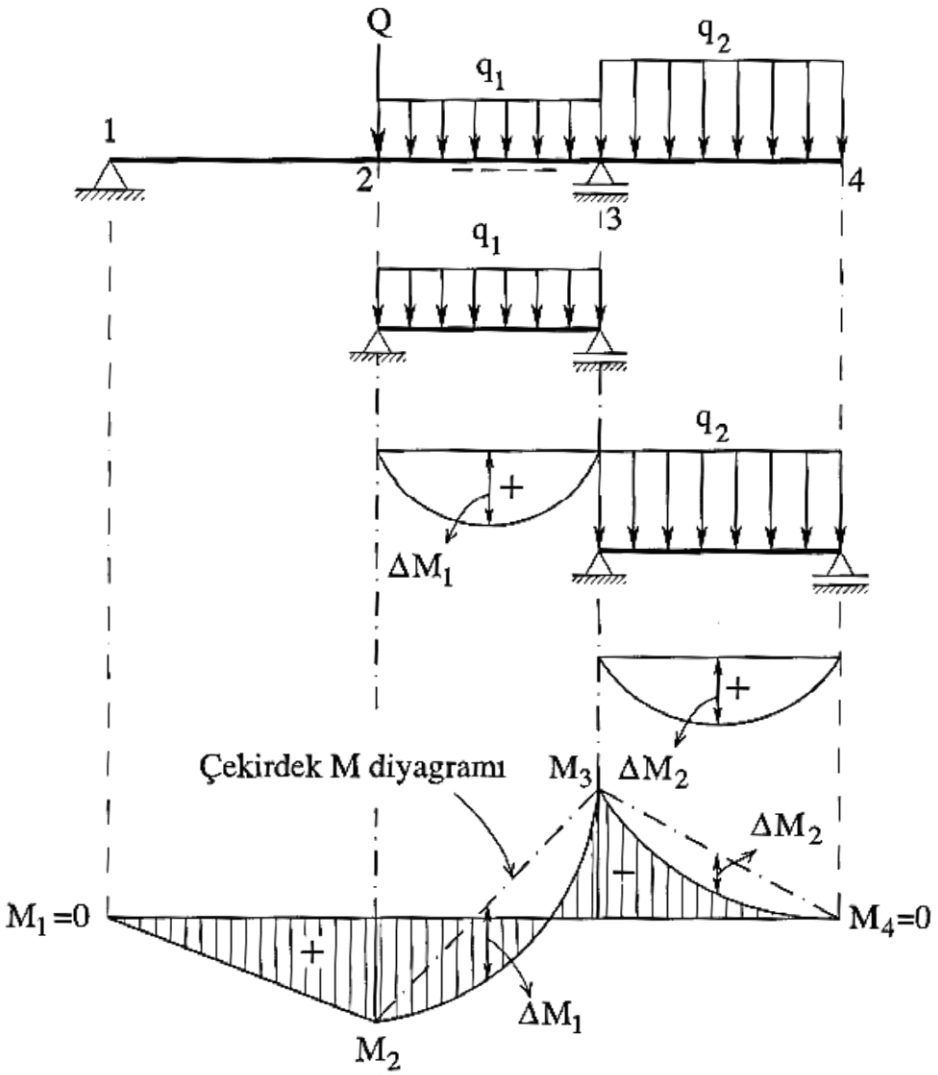


Uygulama:3)



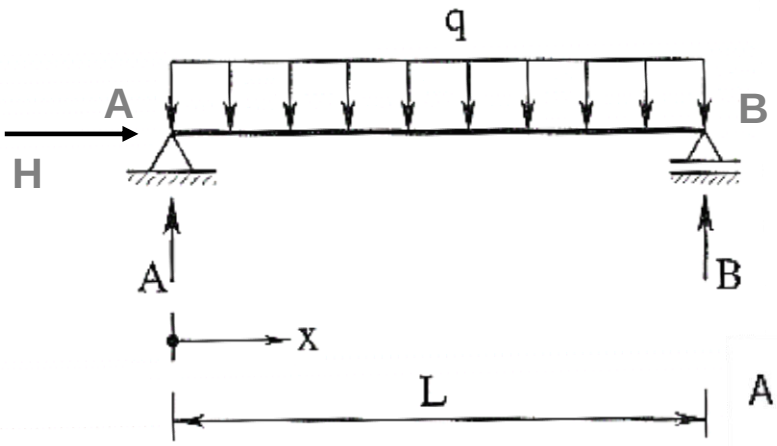
Sonuç : Sistemin (n) ve (n+1) gibi ardışık iki kesitindeki eğilme momentleri belirli iken; bu iki kesit arasındaki eğilme momenti diyagramının çizilebilmesi için (n) ve (n+1) kesitlerindeki eğilme momentlerini ordinat olarak almak suretiyle çizilen doğrusal diyagrama (çekirdek moment diyagramı), (n ~ n+1) açıklıklı basit kirişte q(x) yükünden oluşan eğilme momenti diyagramı cebrik olarak eklenir.

Uygulama



Basit Kirişlerde Çeşitli Yüklerden Meydana Gelen Kesit Tesirleri ve w Sayıları

Düzgün yayılı yük



H=0

$$A = B = \frac{qL}{2}$$

$$M(x) = Ax - \frac{qx^2}{2} = \frac{qL}{2}x - q\frac{x^2}{2}$$

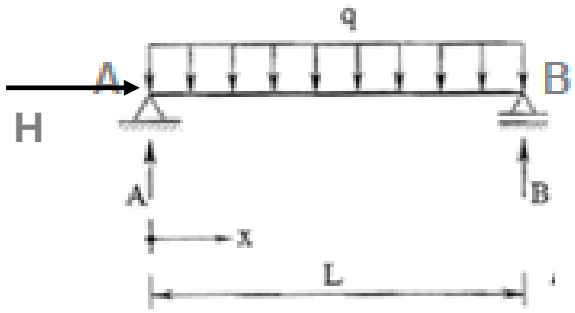
$$M(x) = \frac{qL^2}{2}\left(\frac{x}{L} - \frac{x^2}{L^2}\right)$$

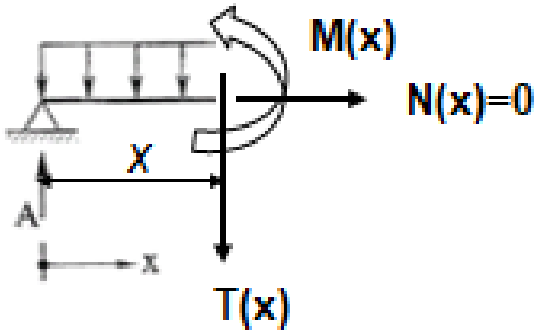
$$w_R = \frac{x}{L} - \frac{x^2}{L^2} \Rightarrow M(x) = w_R \frac{qL^2}{2}$$

ξ	0.25	0.50	0.75
ω_R	0.1875	0.25	0.1875

$$\xi = \frac{x}{L}$$

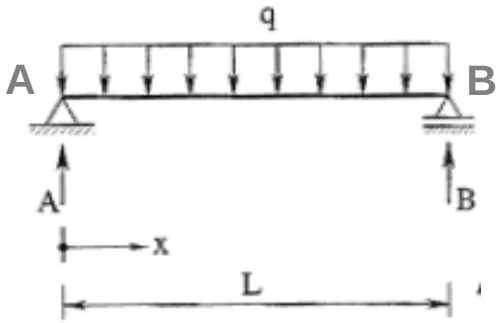
$$\omega_R = \xi - \xi^2$$





$$\sum x = 0 \;\rightarrow\; H = 0\,, \qquad \sum M_B = 0 \;\rightarrow AL - qL\frac{L}{2} = 0 \;\; A = \frac{qL}{2}$$

$$\sum y = 0 \;\rightarrow A + B = qL \rightarrow\; B = qL - \frac{qL}{2} \;\rightarrow B = \frac{qL}{2}$$



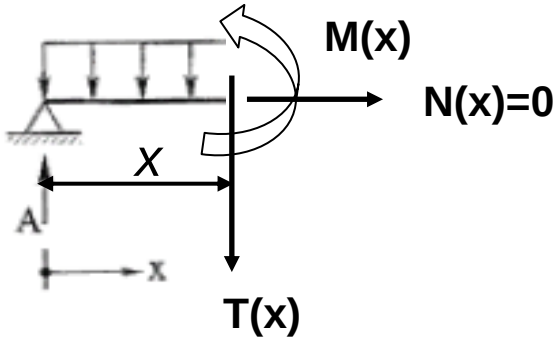
$$A = \frac{qL}{2}$$

$$B = \frac{qL}{2}$$

$$\sum M_A = 0$$

$$\frac{qL}{2}x - \frac{qx^2}{2} - M(x) = 0$$

$$M(x) = \frac{qL}{2}x - \frac{qx^2}{2}$$



$$\xi = \frac{x}{L}$$

$$M(x) = \frac{q\,L^2}{2}\underbrace{(\xi - \xi^2)}_{\omega_R}$$

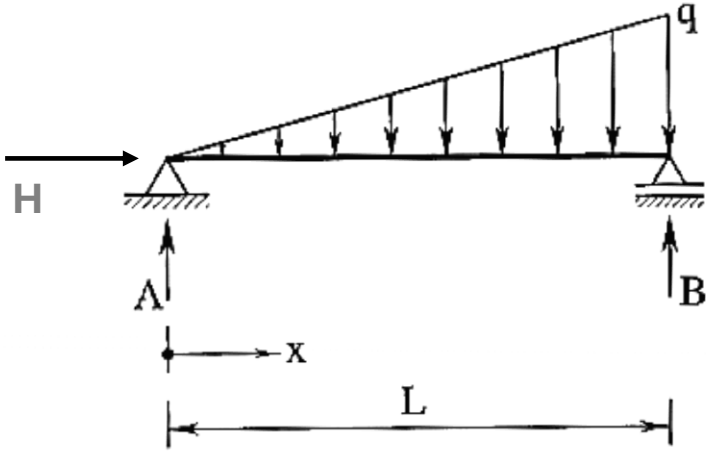
$$M\left(\frac{L}{2}\right) = \frac{qL^2}{8}$$

ξ	0.25	0.50	0.75
ω_R	0.1875	0.25	0.1875

$$\sum X = 0 \rightarrow N(x) = 0$$

$$\sum Y = 0 \rightarrow T(x) = \frac{qL}{2} - qx$$

Üçgen yayılı yük



$$H = 0$$

$$A = \frac{qL}{6} \quad , \quad B = \frac{qL}{3}$$

$$M(x) = \frac{qL^2}{6}\left(\frac{x}{L} - \frac{x^3}{L^3}\right)$$

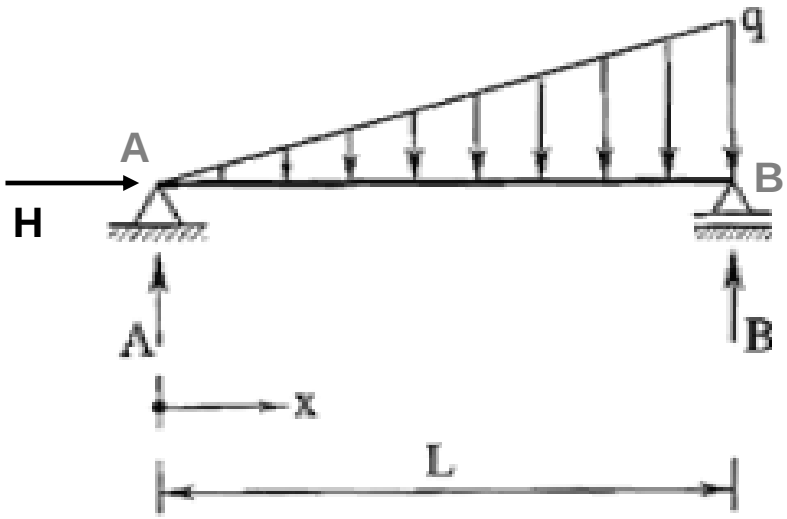
$$w_D = \frac{x}{L} - \frac{x^3}{L^3} \Rightarrow M(x) = w_D \frac{qL^2}{6}$$

ξ	0.25	0.50	0.75
ω_D	0.2344	0.3750	0.3281

$$\xi = \frac{x}{L}$$

$$\omega_D = \xi - \xi^2$$

Not : Üçgen yayılı yükün q ordinatının ters konumda olması halinde tablodaki değerler tersinden gözönüne alınacaktır.

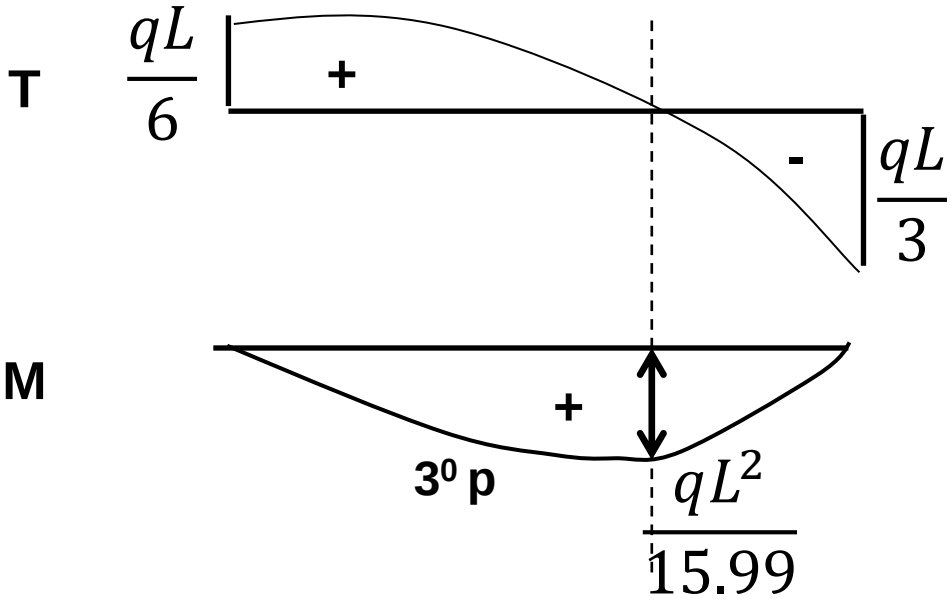


$$H = 0$$

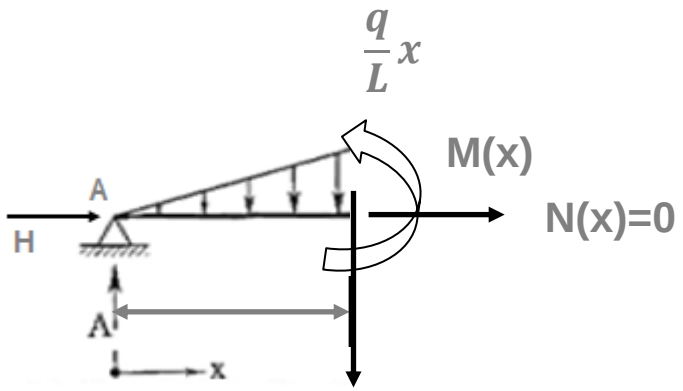
$$\sum M_B = 0 \rightarrow A = \frac{qL}{6}$$

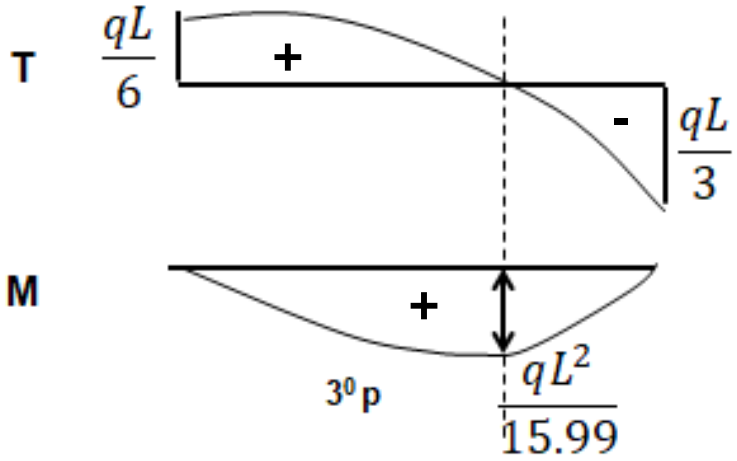
$$AL - \frac{qL}{2}\frac{L}{3} = 0$$

$$A = \frac{qL}{6}$$



$$\sum y = 0 \rightarrow B = \frac{qL}{3}$$





$$N(x) = 0$$

$$T(x) = \frac{qL}{6} - \frac{q}{L}x\frac{x}{2}$$

$$T(x) = \frac{qL}{l}(1 - \frac{3x^2}{L^2})$$

$$M(x) = \frac{qL}{6}x - \frac{q}{L}x\ \frac{x}{2}\frac{x}{3}$$

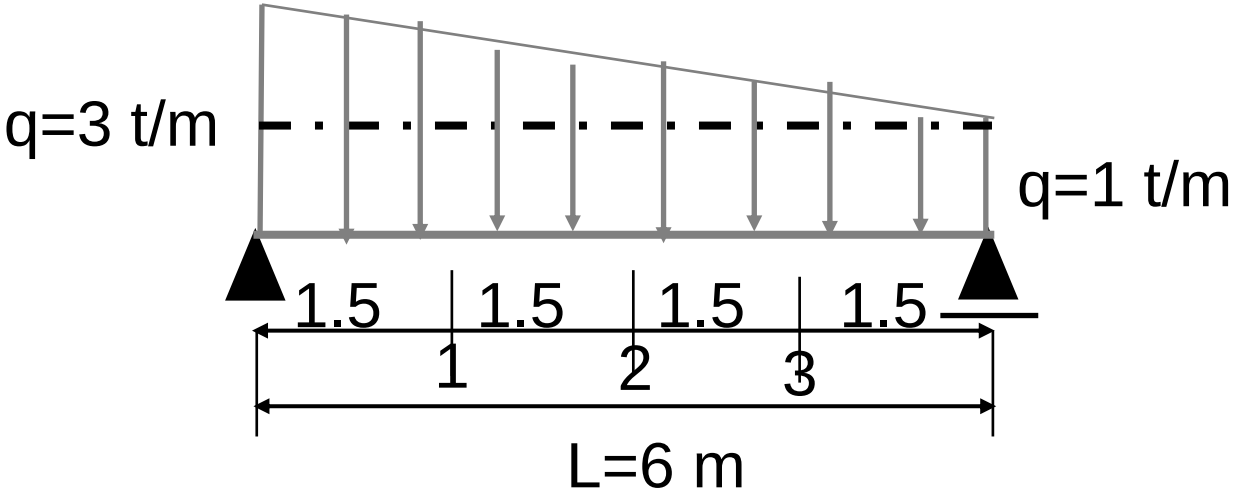
$$M(x) = \frac{qL^2}{6}(\frac{x}{L} - \frac{x^3}{L^3})$$

ξ	0.25	0.50	0.75
ω_D	0.2344	0.3750	0.3281

$$\frac{x}{L} = \xi \quad M(x) = \frac{qL^2}{6}(\underbrace{\xi - \xi^3}_{\omega_D})$$

$$M(x) = \frac{qL^2}{6}\omega_D$$

Örnek 1/4 noktalarında moment değerlerinin bulunuz



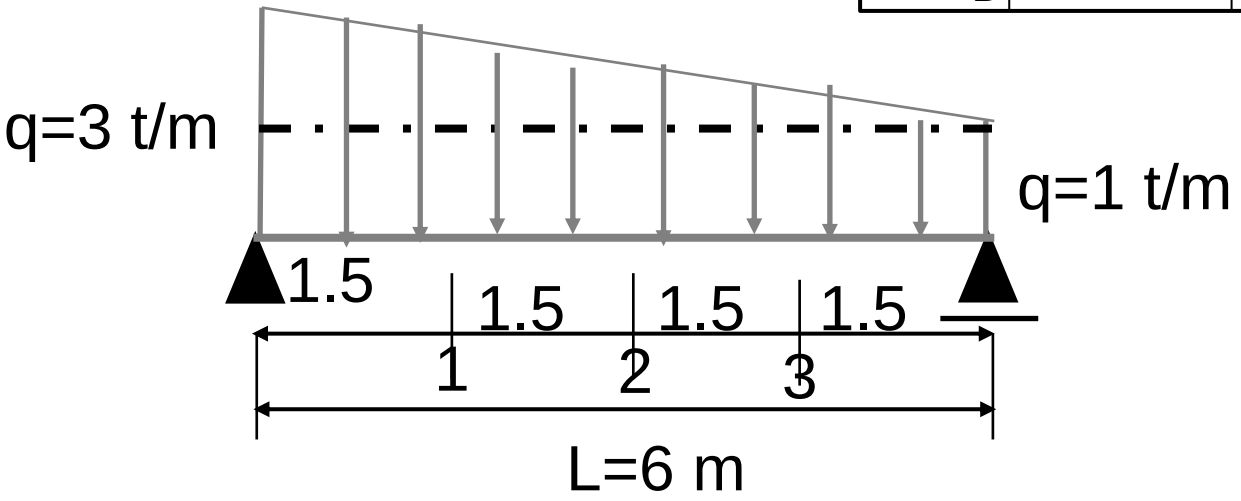
$$M(x) = \frac{qL^2}{2}\omega_R \text{ üniform yük} \quad M(x) = \frac{1.6^2}{2}\omega_R = 18\omega_R$$

$$M'_1 = 18 * 0.1875 = 3.38\ tm$$
$$M'_2 = 18 * 0.25 = 4.5\ tm$$
$$M'_3 = 18 * 0.1875 = 3.38\ tm$$

ξ	0.25	0.50	0.75
ω_R	0.1875	0.25	0.1875

1/4 noktalarında moment değerlerinin bulunuz

ξ	0.25	0.50	0.75
ω_D	0.2344	0.3750	0.3281



$$M(x) = \frac{qL^2}{6}\omega_D \text{ üç}gen\ y\text{ük} \qquad M(x) = \frac{2.6^2}{6}\omega_D = 12\omega_D$$

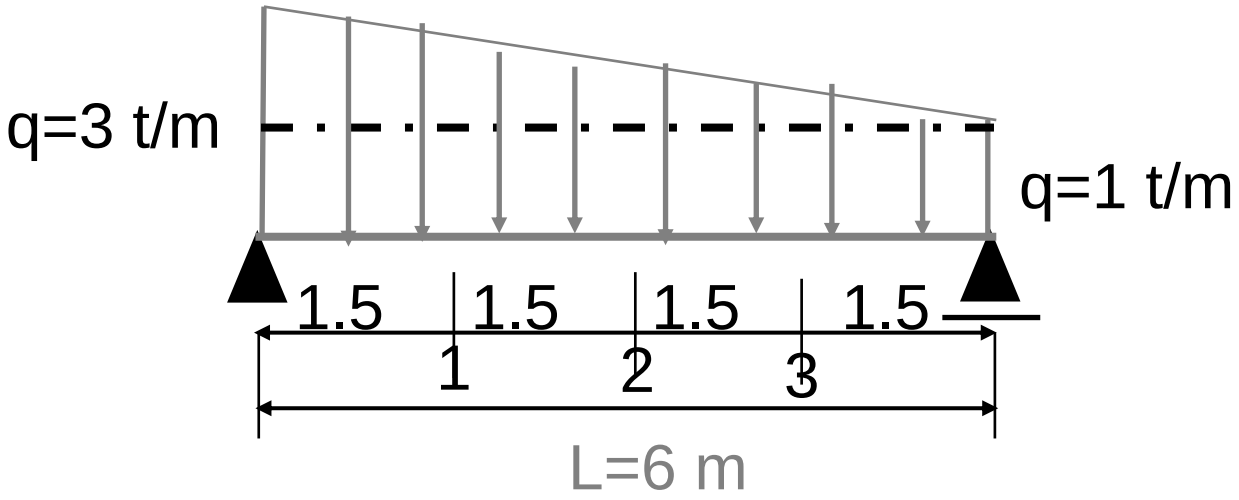
$M''_1 = 12 * 0.3281 = 3.94\ tm$

$M''_2 = 12 * 0.3750 = 4.5\ tm$

$M''_3 = 12 * 0.2344 = 2.81\ tm$

Üçgen yayılı yük ters konumda olduğu için değerler tersinden alınacaktır. (ω_D)

1/4 noktalarında moment değerlerinin bulunuz



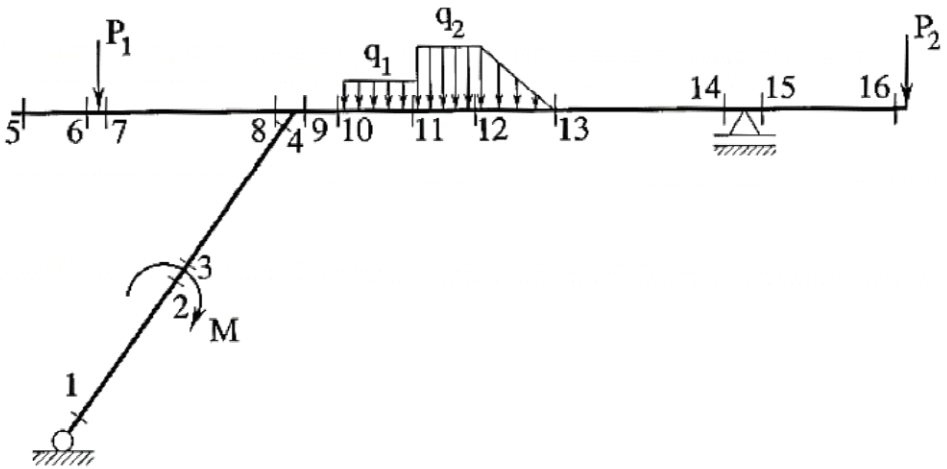
$$M_1 = M'_1 + M''_1 = 3.38 + 3.94 = 7.32\ tm$$
$$M_2 = M'_2 + M''_2 = 4.50 + 4.50 = 9.0\ tm$$
$$M_3 = M'_3 + M''_3 = 3.38 + 2.81 = 6.19\ tm$$

Kritik Kesitler

Diyagramların çizilebilmesi için kesit tesirlerinin hesaplanması gereken kesitlerdir. Bunlar;

- Sistemin uç noktaları
- Yüklerin başlangıç ve bitim noktaları
- Yüklerin şekil ve değer değiştirdiği noktalar
- Tekil yüklerin ve tekil momentlerin her iki yanındaki noktalar
- Mesnetlerin yanındaki noktalar
- Düğüm noktalarına birleşen çubukların uç noktalarıdır.

Uygulama



Kesit Zorları Diyagramlarının Çiziminde İzlenen Yol

1- Mesnet tepkileri hesaplanır.

2- Kritik kesitler belirlenir.

3- Kritik kesitlerdeki M, N, T kesit zorları hesaplanır. (Gerekirse Teorem-1 den yararlanılabilir.)

4- N, T diyagramları çizilir. (T diyagramının çiziminde $\frac{dT}{dx} = -q$ bağıntısından yararlanılır.)

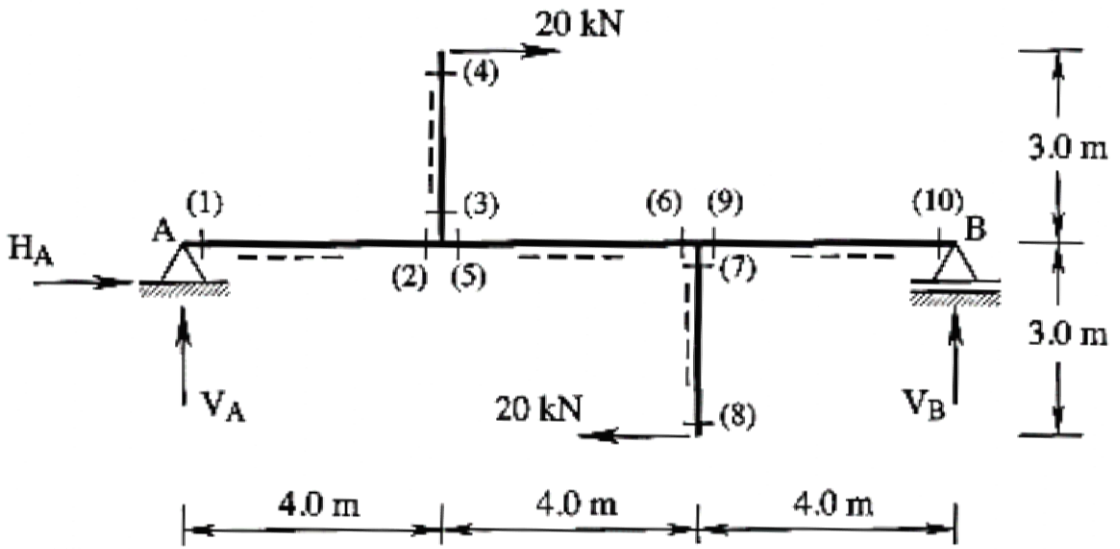
5- Çekirdek M diyagramı çizilir.

6- M diyagramı tamamlanır. (Teorem-2 ve (w) tablolarından yararlanılır.)

7- Maksimum eğilme momentleri hesaplanır. $\left(\frac{dM}{dx} = T = 0\right)$

Çözümlü Problemler

PROBLEM 4.1. Şekil 4.1 de geometrisi ve yükleme durumu verilen basit kirişin M, N, T diyagramlarını çizinlz.



Basit kirişin verilen yüklemeden dolayı oluşan mesnet tepkileri hesaplanır.

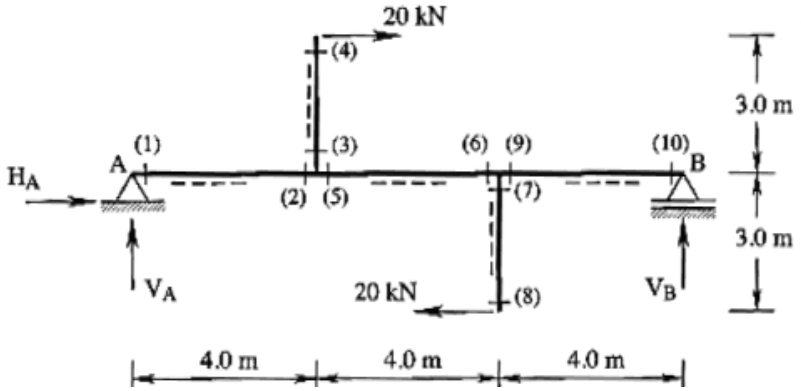
$(\circlearrowleft+)\Sigma M_A=0 \Rightarrow 20\times3+20\times3-12V_B=0 \rightarrow V_B=10\ \text{kN}$

$(\circlearrowleft+)\Sigma M_B=0 \Rightarrow 12V_A+20\times3+20\times3=0 \rightarrow V_A=-10\ \text{kN}$

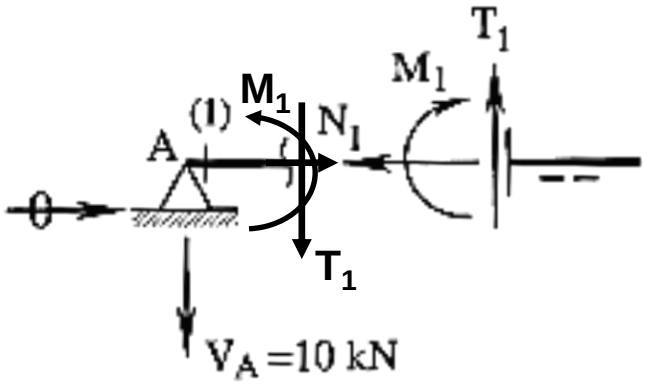
Kontrol: $(\uparrow+)\ \Sigma F_Y=0 \quad \Rightarrow V_A+V_B=0$ ✓

$(\rightarrow+)\ \Sigma F_X=0 \quad \Rightarrow H_A=0$

Mesnet tepkileri bulunduktan sonra kritik kesitlerin yerleri belirlenir ve bu kesitlerden sistem kesilerek kesit zorları hesaplanır.



1 nolu kesit:

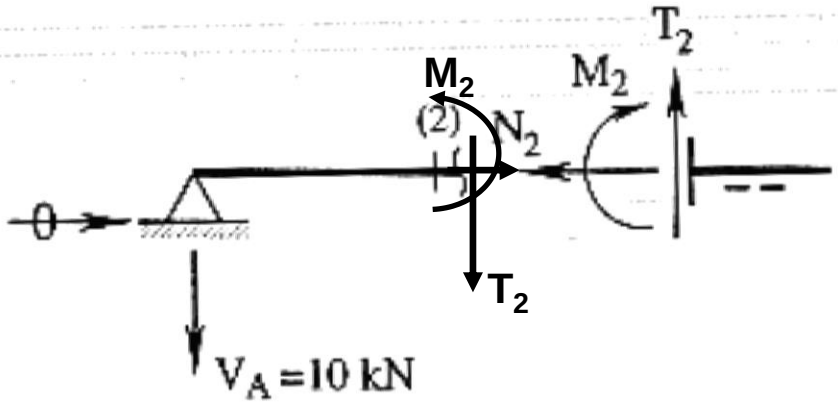


$M_1 = 0$

$N_1 = 0$

$T_1 = -10$ kN

2 nolu kesit:

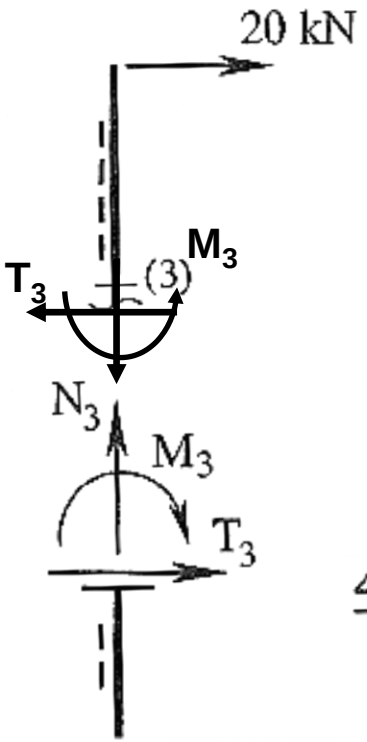


$M_2 = -10 \times 4 = -40$ kNm

$N_2 = 0$

$T_2 = -10$ kN

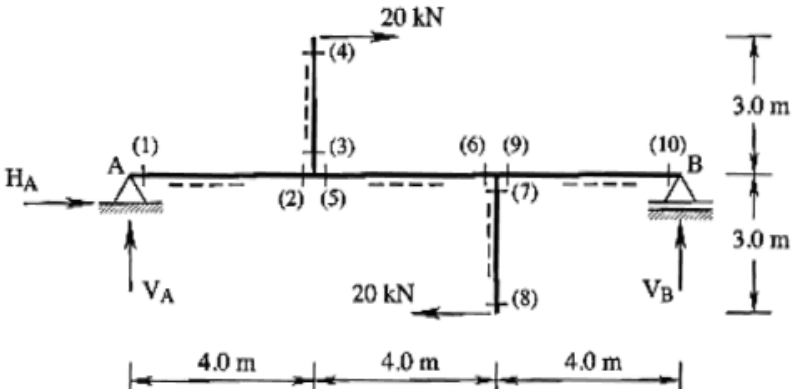
3 nolu kesit:



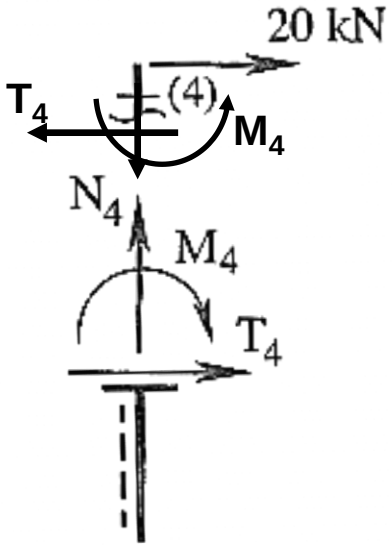
$M_3 = 20 \times 3 = 60$ kNm

$N_3 = 0$

$T_3 = 20$ kN



4 nolu kesit:

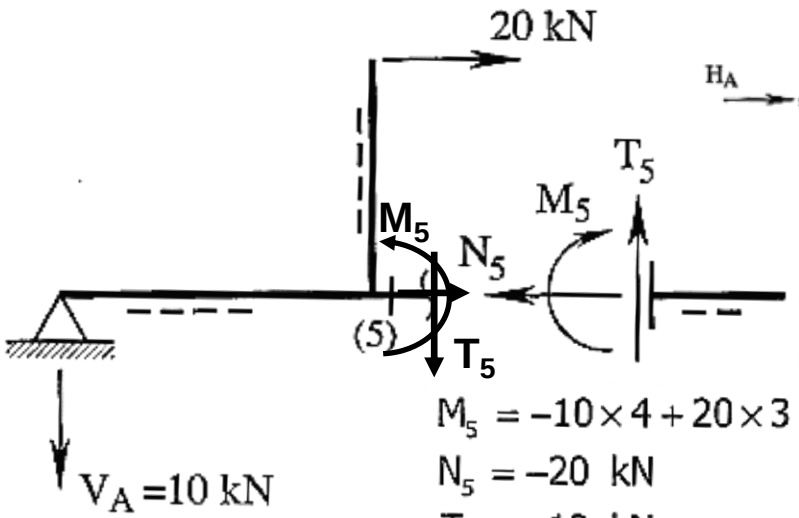


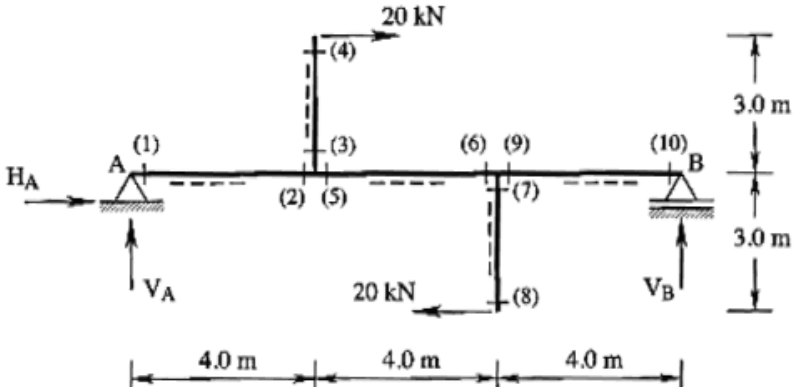
$M_4 = 0$

$N_4 = 0$

$T_4 = 20$ kN

5 nolu kesit:





$M_5 = -10 \times 4 + 20 \times 3 = 20$ kNm

$N_5 = -20$ kN

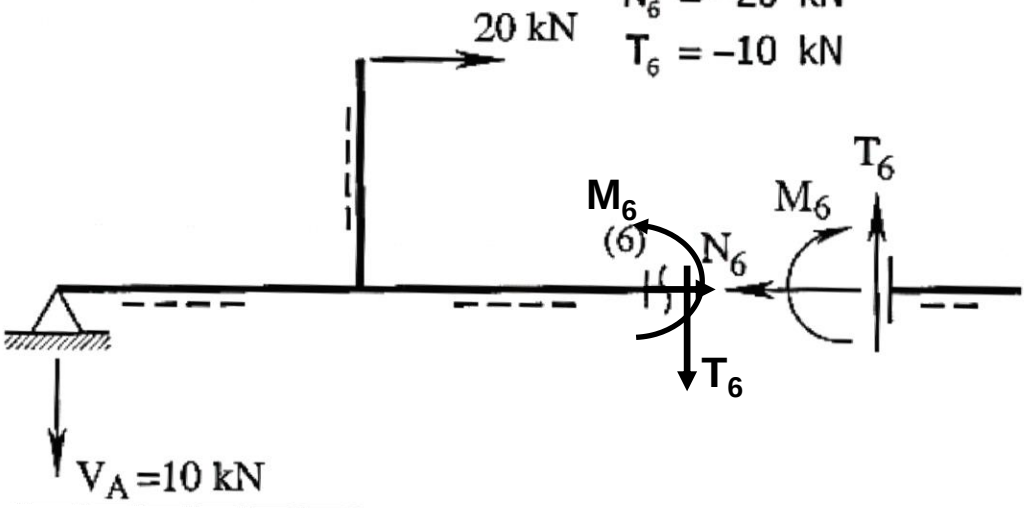
$T_5 = -10$ kN

6 nolu kesit:

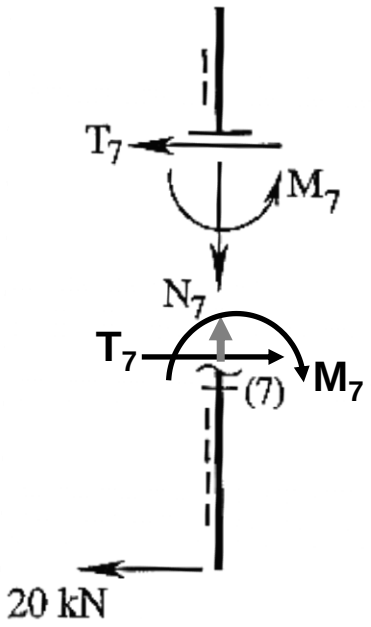
$M_6 = -10 \times 8 + 20 \times 3 = -20$ kNm

$N_6 = -20$ kN

$T_6 = -10$ kN



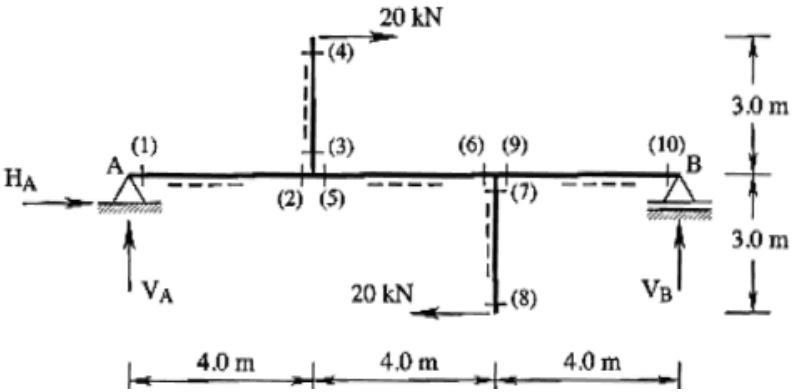
7 nolu kesit:



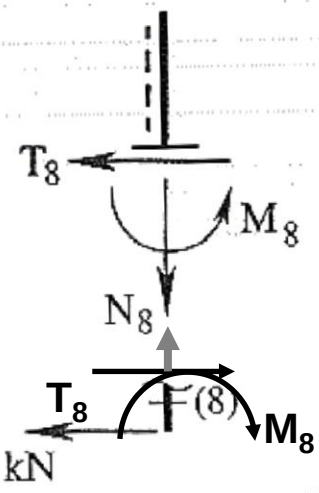
$M_7 = -20 \times 3 = -60$ kNm

$N_7 = 0$

$T_7 = 20$ kN



8 nolu kesit:

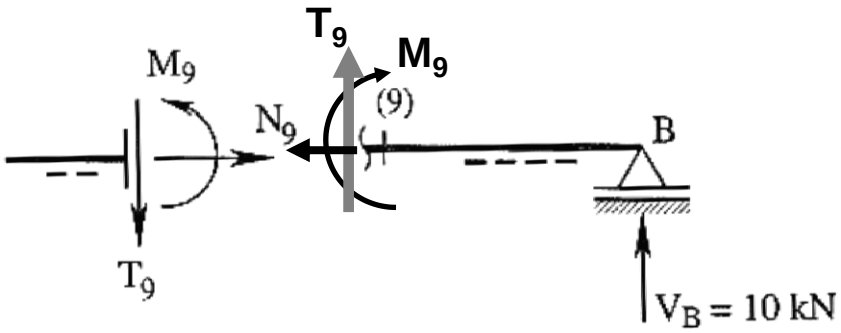


$M_8 = 0$

$N_8 = 0$

$T_8 = 20$ kN

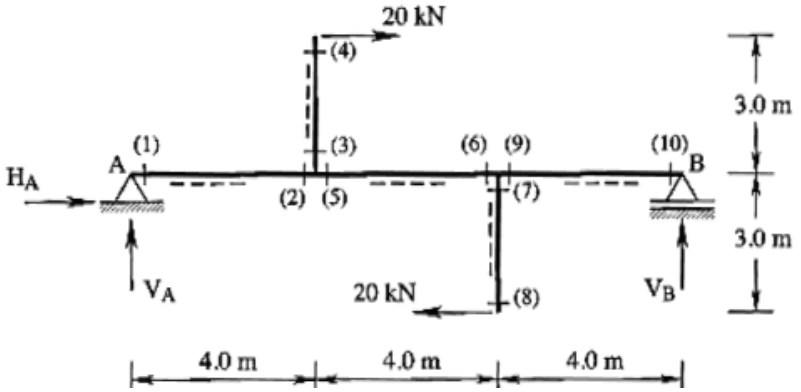
9 nolu kesit:



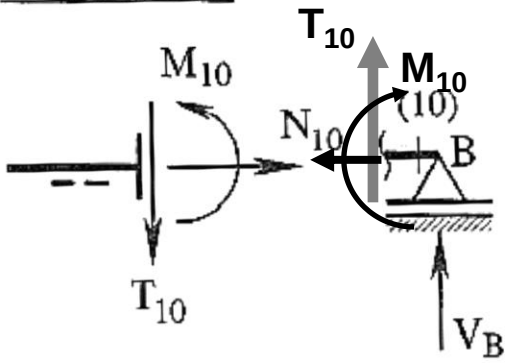
$M_9 = 10 \times 4 = 40$ kNm

$N_9 = 0$

$T_9 = -10$ kN



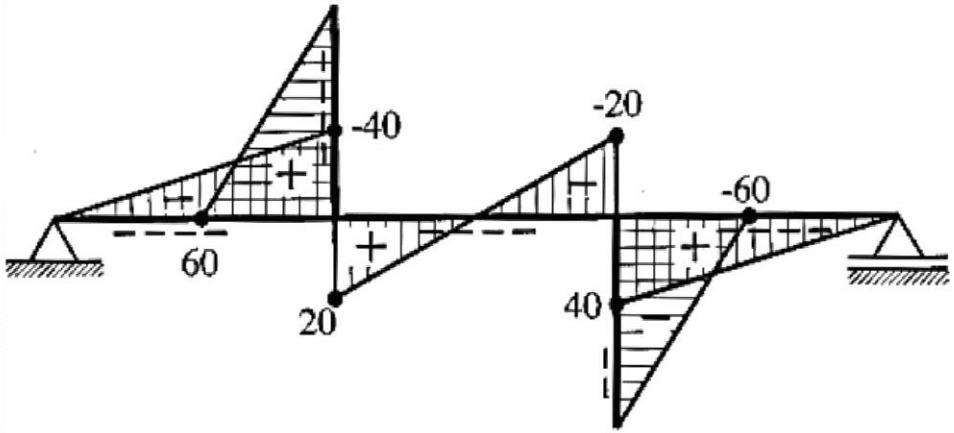
10 nolu kesit:



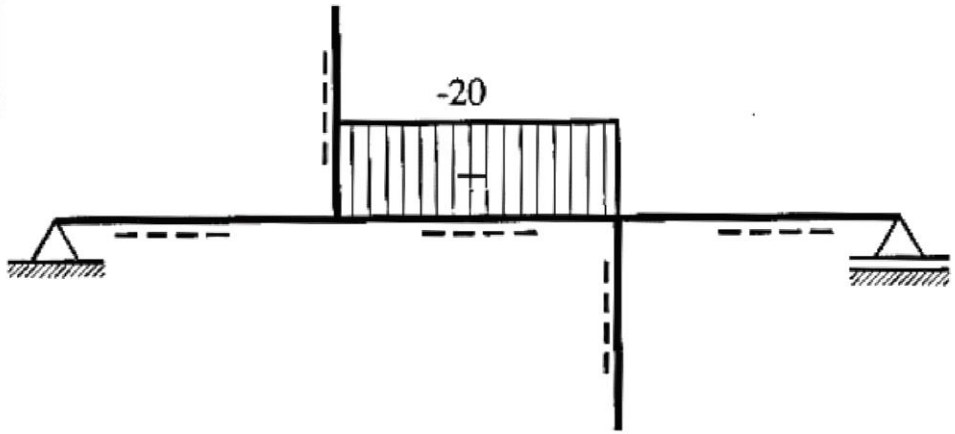
$M_{10} = 0$

$N_{10} = 0$

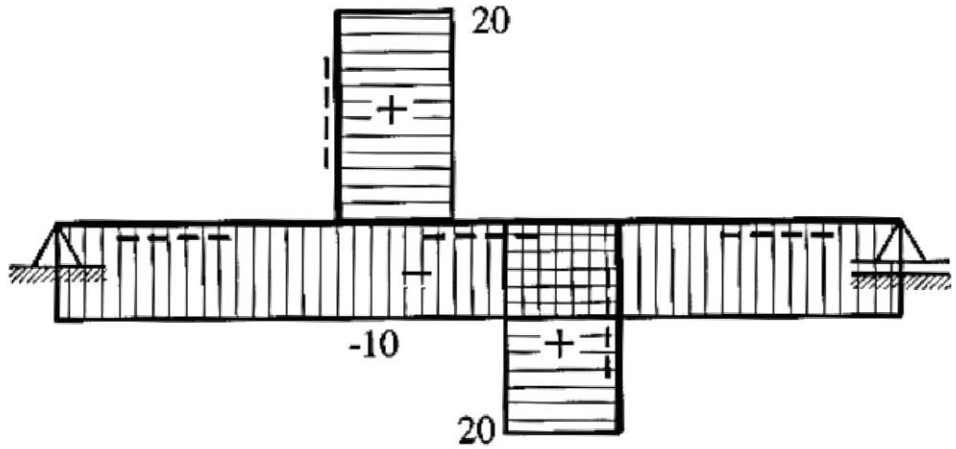
$T_{10} = -10$ kN



(M^{kNm})



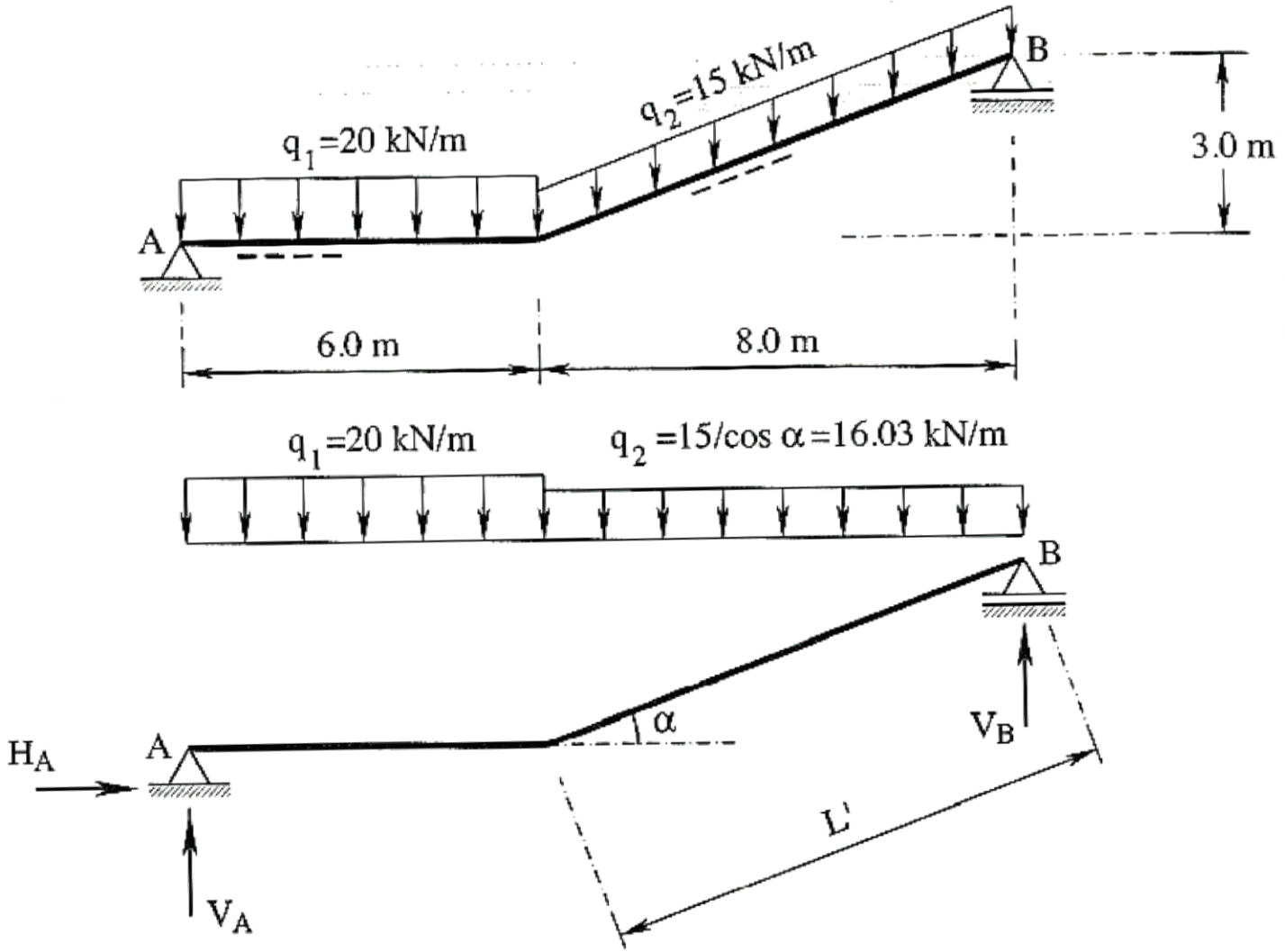
(N^{kN})



(T^{kN})

PROBLEM 4.2.

Şekil 4.2 deki basit kirişin üzerine etkiyen dış yüklerden oluşan mesnet tepkilerini hesaplayınız. Kesme kuvveti (T), eğilme momenti (M) ve normal kuvvet (N) diyagramlarını çiziniz.



q_2 yayılı yükünü, yatay düzlemde yayılı yüke dönüştürmek için α açısının değeri hesaplanır.

$$\tan\alpha = \frac{3}{8} \rightarrow \alpha = 20.556^\circ \rightarrow \mathrm{Sin}\,\alpha = 0.351\ ,\ \mathrm{Cos}\,\alpha = 0.936$$

Yatay düzleme etkiyen yayılı yükün değeri

$$q_2' = \frac{q_2}{\mathrm{Cos}\,\alpha} = \frac{15}{0.936} = 16.03\ \mathrm{kN/m}$$

olarak hesaplanır.

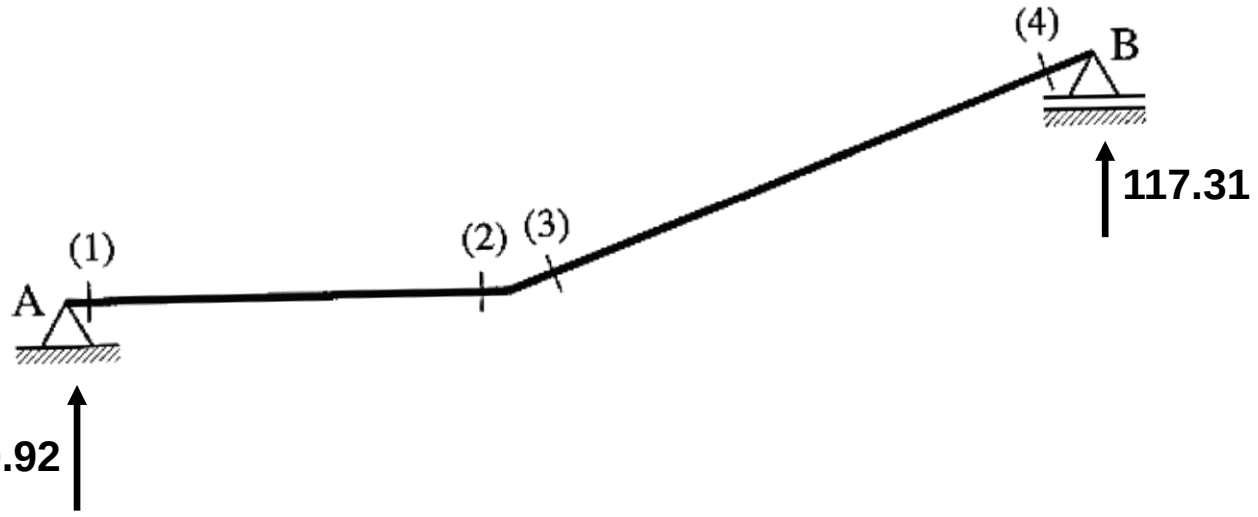
$$(\rightarrow +)\ \Sigma F_x = 0 \Rightarrow H_A = 0$$

$$(\circlearrowleft +)\ \Sigma M_A = 0 \Rightarrow 20\times 6\times 3 + 16.03\times 8\times 10 - 14\,V_B = 0 \rightarrow V_B = 117.31\,\mathrm{kN}$$

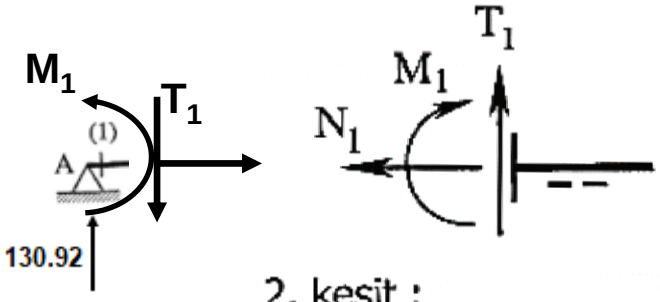
$$(\circlearrowleft +)\ \Sigma M_B = 0 \Rightarrow 14\,V_A - 20\times 6\times 11 - 16.03\times 8\times 4 = 0 \rightarrow V_A = 130.92\,\mathrm{kN}$$

Kontrol : $(\uparrow +)\ \Sigma F_Y = 0 \Rightarrow V_A + V_B - 20\times 6 - 16.03\times 8 = 0$ ✔

Kritik kesitler belirlenir



1. kesit :

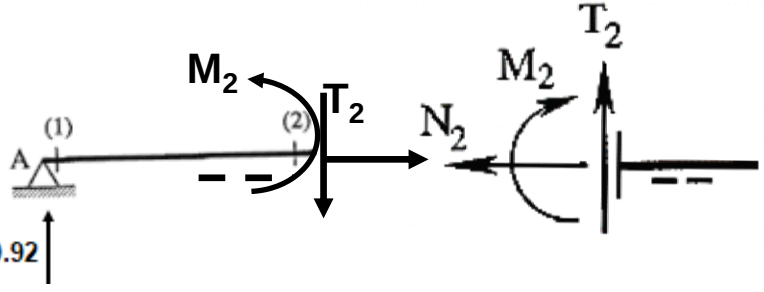


$M_1 = 0$

$N_1 = 0$

$T_1 = 130.92 \text{ kN}$

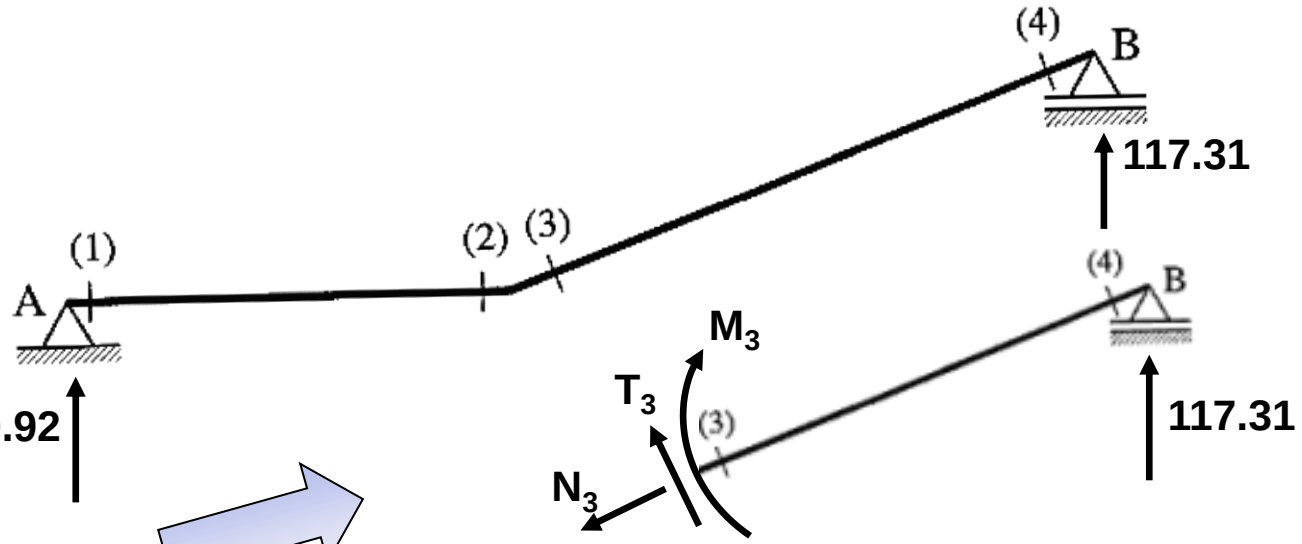
2. kesit :



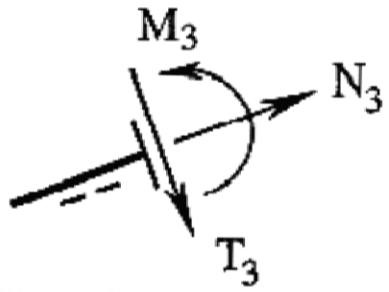
$M_2 = 130.92 \times 6 - 20 \times 6 \times 3 = 425.52 \text{ kNm}$

$N_2 = 0$

$T_2 = 130.92 - 20 \times 6 = 10.92 \text{ kN}$



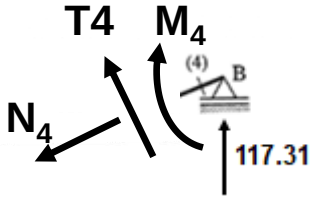
3. kesit :



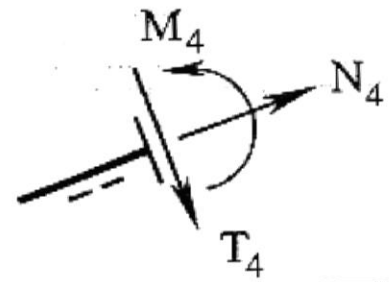
$$M_3 = 117.31 \times 8 - 16.03 \times 8 \times 4 = 425.52 \text{ kNm}$$

$$N_3 = (117.31 - 16.03 \times 8)\,\text{Sin}\,\alpha = -3.84 \text{ kN}$$

$$T_3 = (16.03 \times 8 - 117.31)\,\text{Cos}\,\alpha = 10.23 \text{ kN}$$



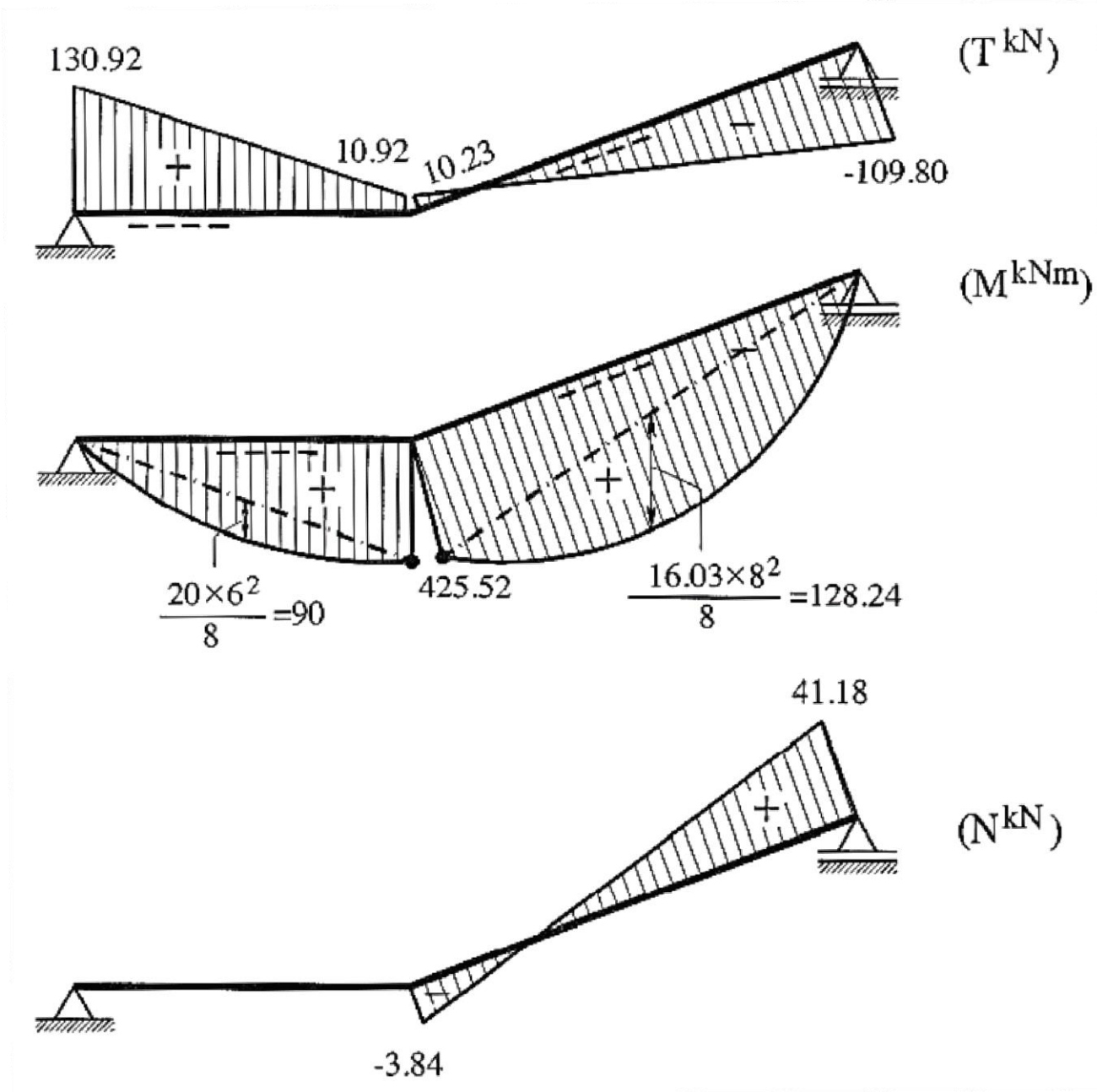
4. kesit :



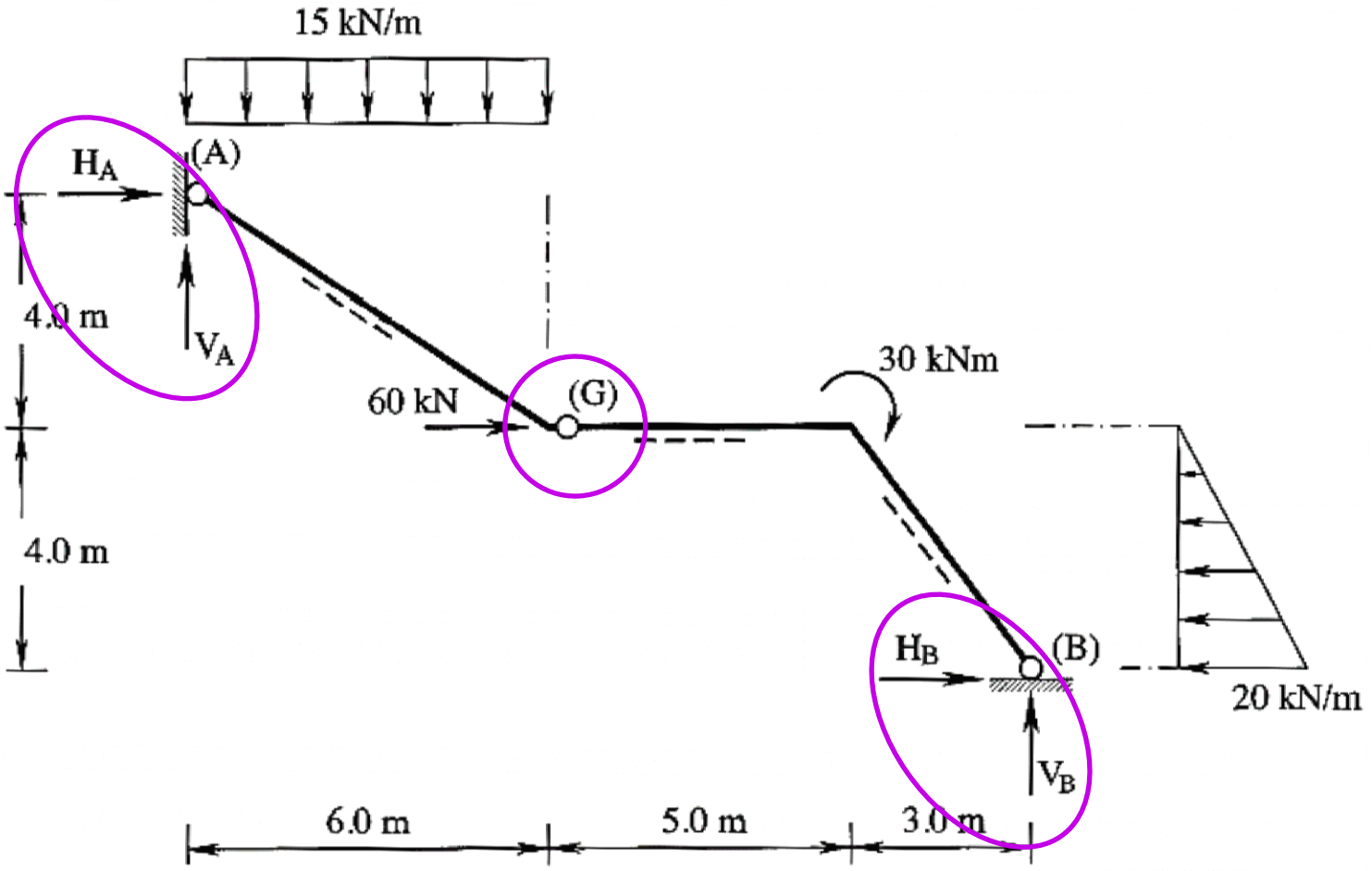
$$M_4 = 0$$

$$N_4 = 117.31 \times \text{Sin}\,\alpha = 41.18 \text{ kN}$$

$$T_4 = -117.31 \times \text{Cos}\,\alpha = -109.80 \text{ kN}$$


130.92
10.92
10.23
-109.80
(T kN)
(M kNm)
20×6²/8 =90
425.52
16.03×8²/8 =128.24
41.18
-3.84
(N kN)

PROBLEM 4.3. Şekil 4.3 de verilen sistemin M, N, T diyagramlarını çiziniz.



4 mesnet tepkisi= 3 denge denklemi + 1 mafsal şartı

$$(\circlearrowleft+)\ \Sigma M_A=0 \Rightarrow 15\times 6\times 3-60\times 4+30+40\times 6.667-8\times H_B-14\times V_B=0$$

$$\Rightarrow 14\times V_B+8\times H_B=326.68 \quad \textbf{(I)}$$

$$(\circlearrowleft+)\ \Sigma M_{G,sağ}=0 \Rightarrow 30+40\times\frac{8}{3}-4\times H_B-8\times V_B=0$$

$$\Rightarrow 8\times V_B+4\times H_B=136.667 \quad \textbf{(II)}$$

$$8\times H_B+14\times V_B=326.68$$

$$4\times H_B+8\times V_B=136.667$$

$$\Rightarrow V_B=-26.67\ \text{kN} \quad , \quad H_B=87.50\ \text{kN}$$

$$(\circlearrowleft+)\ \Sigma M_B=0 \Rightarrow 14\times V_A+8\times H_A-15\times 6\times 11+60\times 4+30-40\times\frac{4}{3}=0$$

$$\Rightarrow 14\times V_A+8\times H_A=773.333 \quad \textbf{(III)}$$

$$(\circlearrowleft+)\ \Sigma M_{G,sol}=0 \Rightarrow 6\times V_A+4\times H_A-15\times 6\times 3=0$$

$$\Rightarrow 6\times V_A+4\times H_A=270 \quad \textbf{(IV)}$$

$$8\times H_A+14\times V_A=773.333$$

$$4\times H_A+6\times V_A=270$$

$$\Rightarrow V_A=116.67\ \text{kN} \quad , \quad H_A=-107.50\ \text{kN}$$

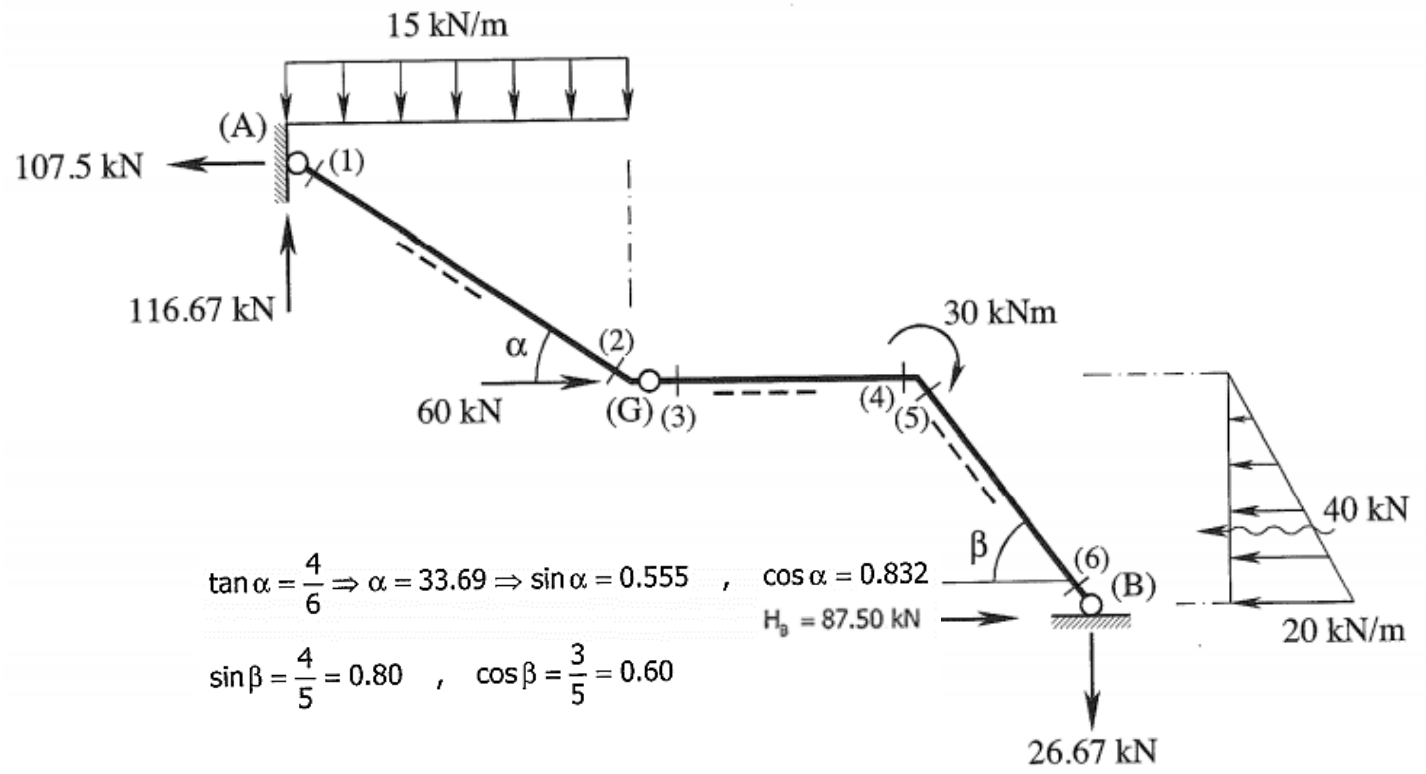
Kontrol-1) (↑+) $\Sigma F_Y = 0$ olmalıdır.

$$\Rightarrow 116.67 - 26.67 - 15 \times 6 = 0$$ ✔

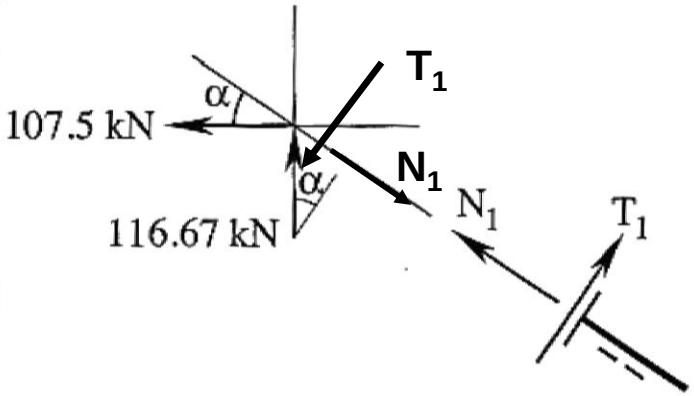
Kontrol-2) (→+) $\Sigma F_X = 0$ olmalıdır.

$$\Rightarrow -107.50 + 87.50 + 60 - 40 = 0$$ ✔

Kritik kesitler tanımlanır ve kesimler yapılarak kesit zoru değerleri hesaplanır.

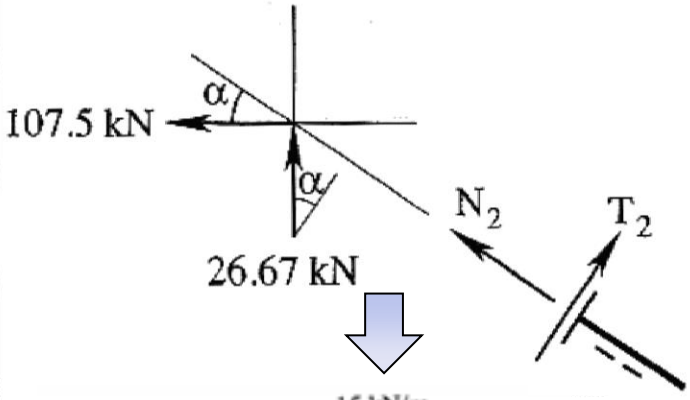


1 nolu kesit:



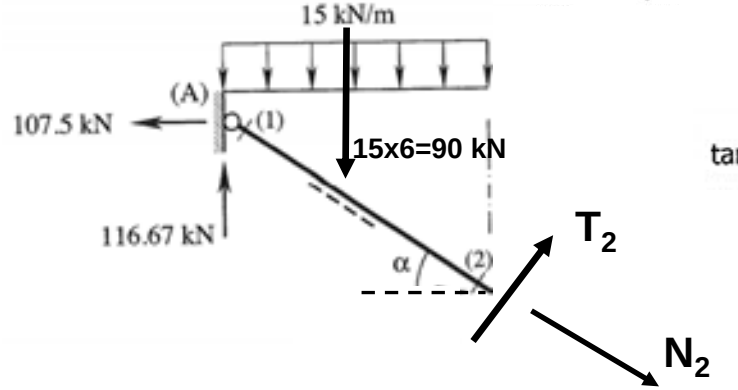
$$M_1 = 0$$
$$N_1 = 107.50 \times 0.832 + 116.67 \times 0.555 = 154.19 \text{ kN}$$
$$T_1 = -107.50 \times 0.555 + 116.67 \times 0.832 = 37.41 \text{ kN}$$

2 nolu kesit:



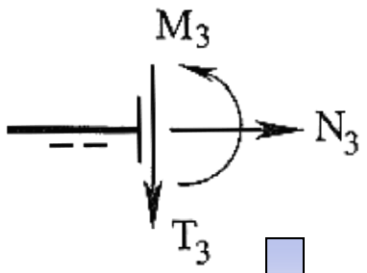
116.67-90=26.67kN

$$M_2 = 0$$
$$N_2 = 107.50 \times 0.832 + 26.67 \times 0.555 = 104.24 \text{ kN}$$
$$T_2 = -107.50 \times 0.555 + 26.67 \times 0.832 = -37.47 \text{ kN}$$



$$\tan\alpha = \frac{4}{6} \Rightarrow \alpha = 33.69 \Rightarrow \sin\alpha = 0.555 \quad , \quad \cos\alpha = 0.832$$

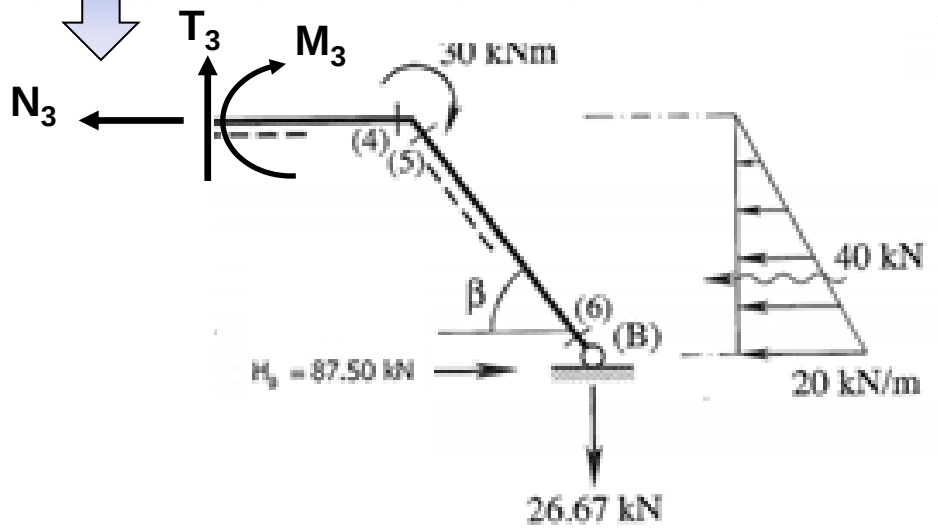
3 nolu kesit:



$M_3 = 0$

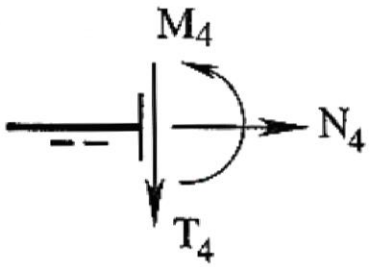
$N_3 = 87.50 - \frac{1}{2} \times 20 \times 4 = 47.50 \text{ kN}$

$T_3 = 26.67 \text{ kN}$



$\sin\beta = \frac{4}{5} = 0.80 \quad , \quad \cos\beta = \frac{3}{5} = 0.60$

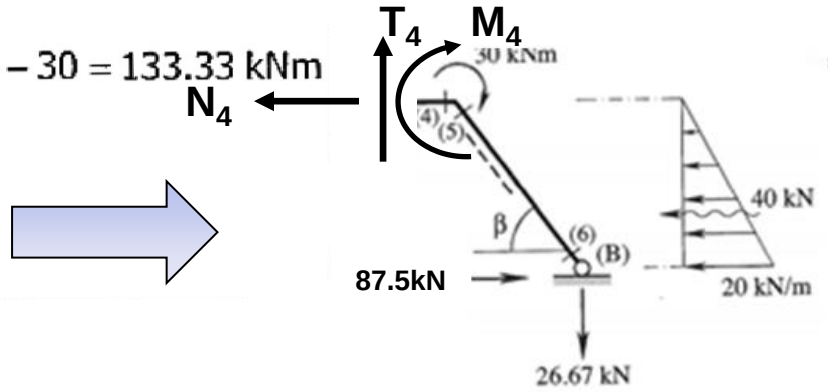
4 nolu kesit:



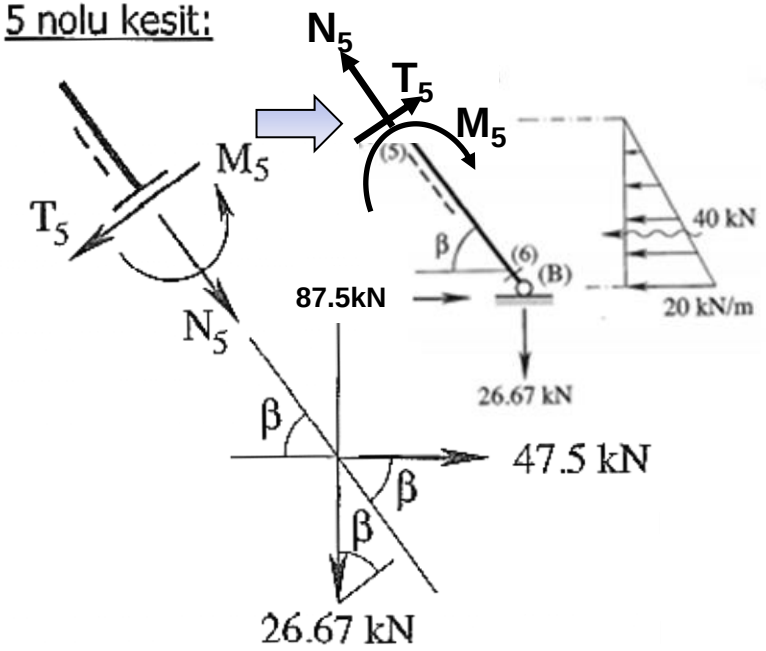
$$M_4 = 87.50\times 4 - 26.67\times 3 - 40\times\frac{8}{3} - 30 = 133.33 \text{ kNm}$$

$$N_4 = N_3 = 47.50 \text{ kN}$$

$$T_4 = T_3 = 26.67 \text{ kN}$$



5 nolu kesit:

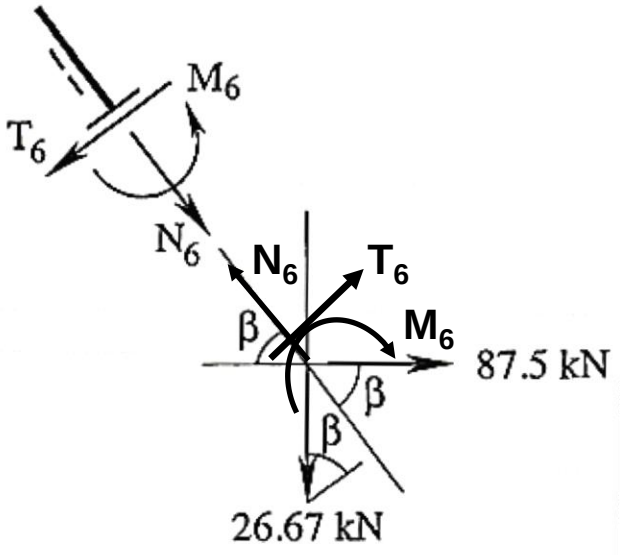


$$M_5 = 87.50\times 4 - 26.67\times 3 - 40\times\frac{8}{3} = 163.33 \text{ kNm}$$

$$N_5 = 47.50\times 0.60 + 26.67\times 0.80 = 49.84 \text{ kN}$$

$$T_5 = -47.50\times 0.80 + 26.67\times 0.60 = -22.00 \text{ kN}$$

6 nolu kesit:

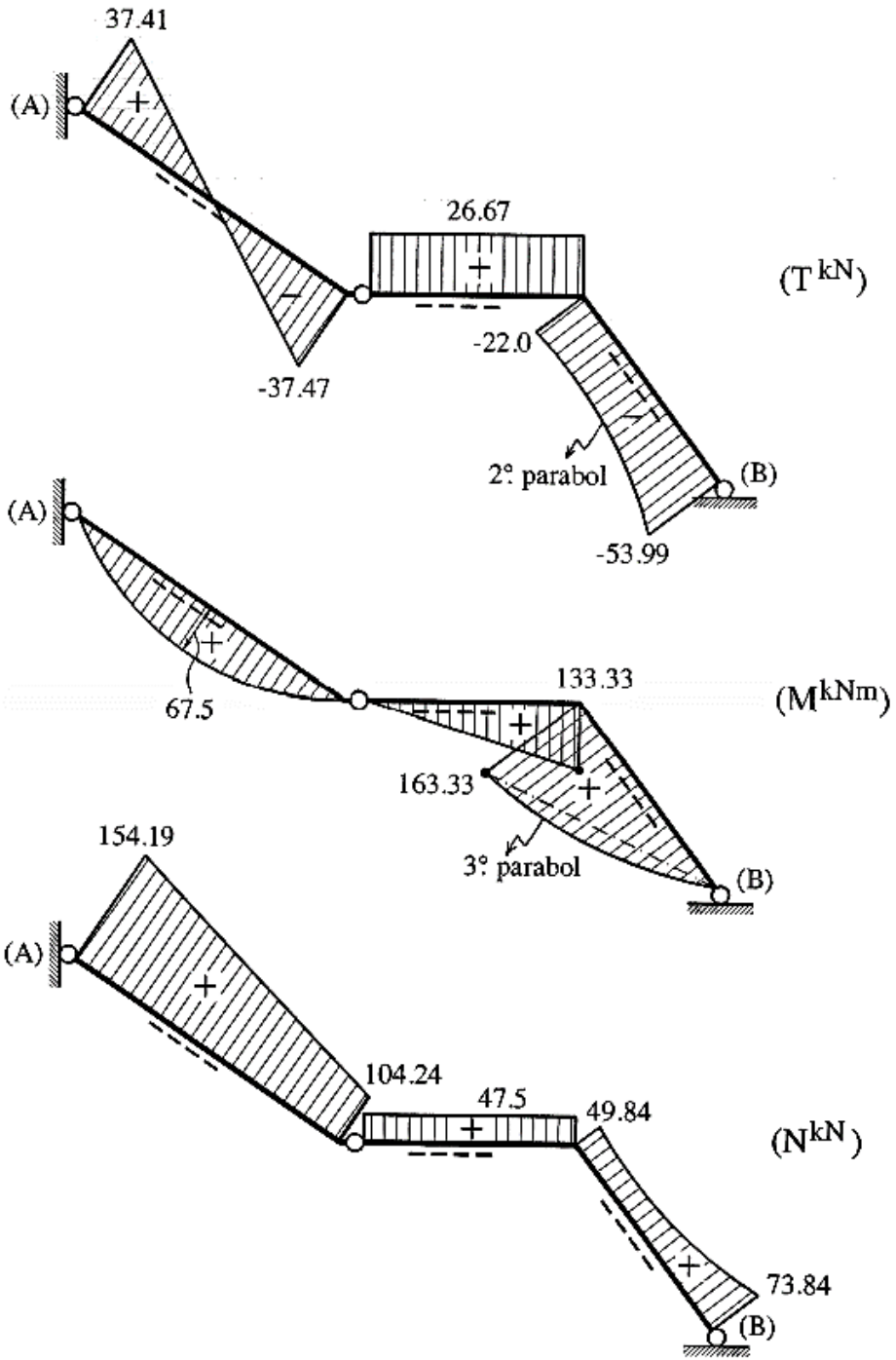


$$M_6 = 0$$

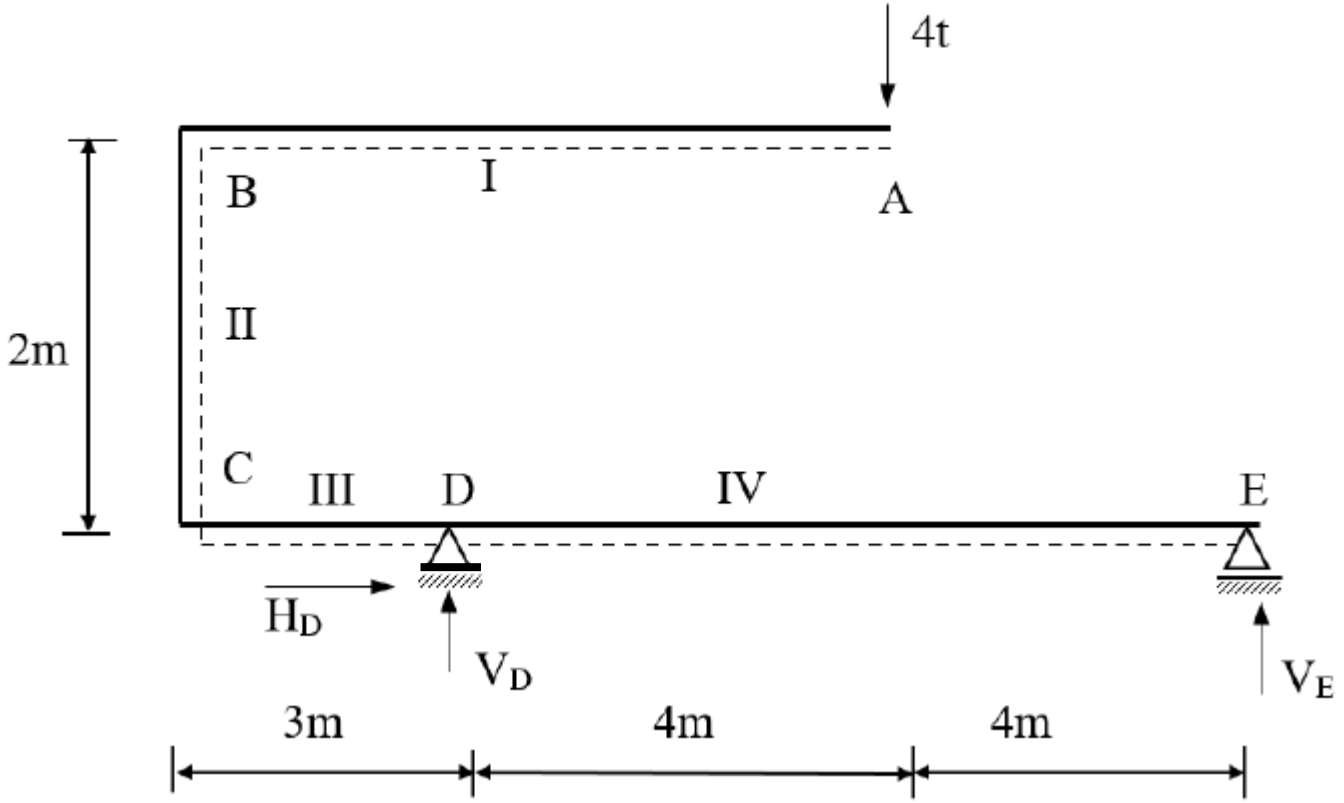
$$N_6 = 87.50\times 0.60 + 26.67\times 0.80 = 73.84 \text{ kN}$$

$$T_6 = -87.50\times 0.80 + 26.67\times 0.60 = -53.99 \text{ kN}$$

$$\sin\beta = \frac{4}{5} = 0.80 \quad , \quad \cos\beta = \frac{3}{5} = 0.60$$


37.41
(A)
26.67
(T kN)
-22.0
-37.47
2° parabol
(B)
-53.99
(A)
133.33
(M kNm)
67.5
163.33
3° parabol
(B)
154.19
(A)
104.24
47.5
49.84
(N kN)
73.84
(B)

PROBLEM 4.4 Şekildeki sistemin M, N, T diyagramlarını çiziniz.

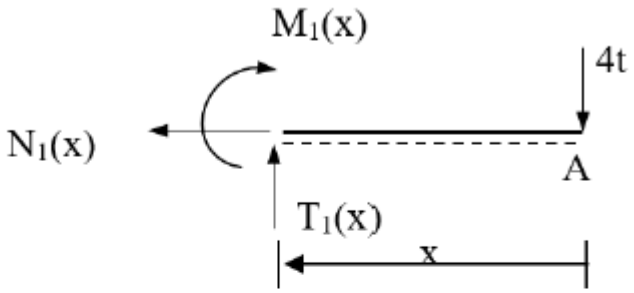


$\Sigma Fx=0 \quad H_D=0$

$\Sigma M_D=0 \quad 4.4-8.V_E=0 \Rightarrow V_E=2t$

$\Sigma Fy=0 \quad V_D+V_E-4=0 \Rightarrow V_D+2-4=0 \Rightarrow V_D=2t$

I bölgesi $0 \le x \le 7m$



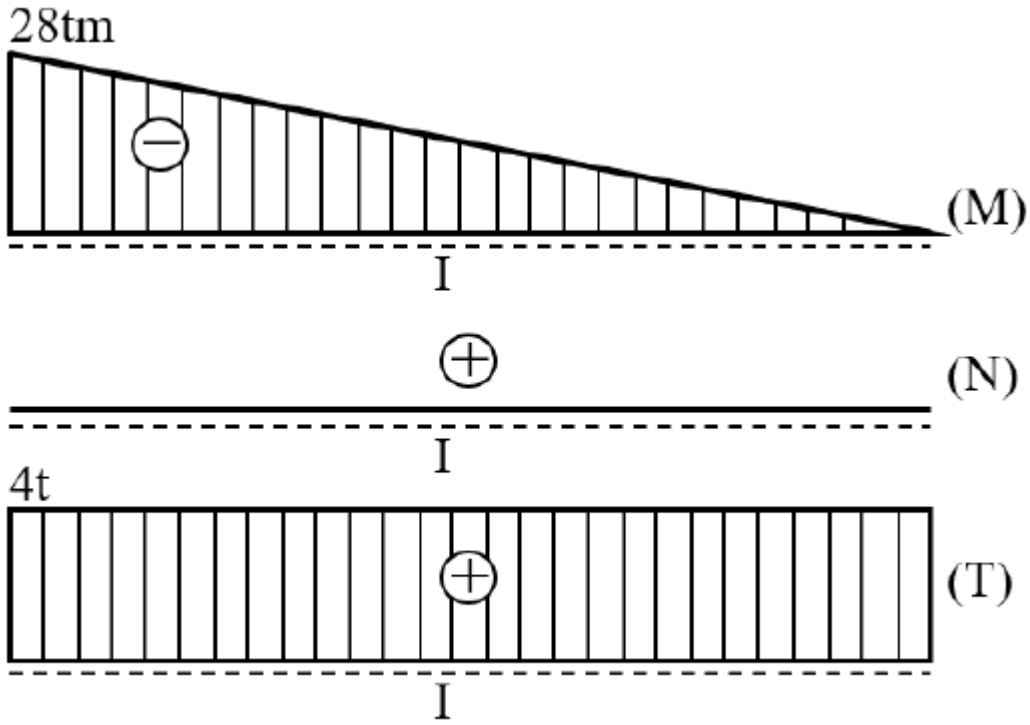
$\Sigma Fx=0$ $N_1(x)=0$

$\Sigma Fy=0$ $T_1(x)-4=0$ $T_1(x)=4t$

$\Sigma M=0$ $M_1(x)+(4)x=0$ $M_1(x)=-4x$

$x=0$ için $M_1(x)=0$

$x=7$ için $M_1(x)=-4(7)=-28tm$

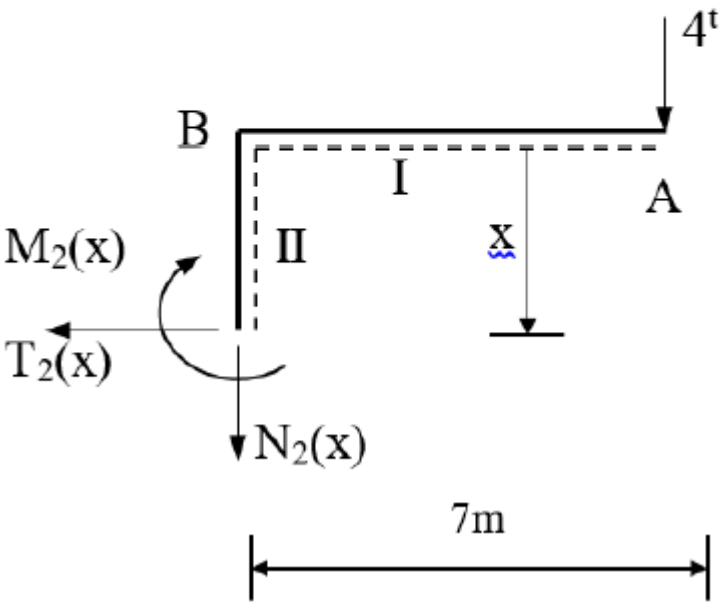


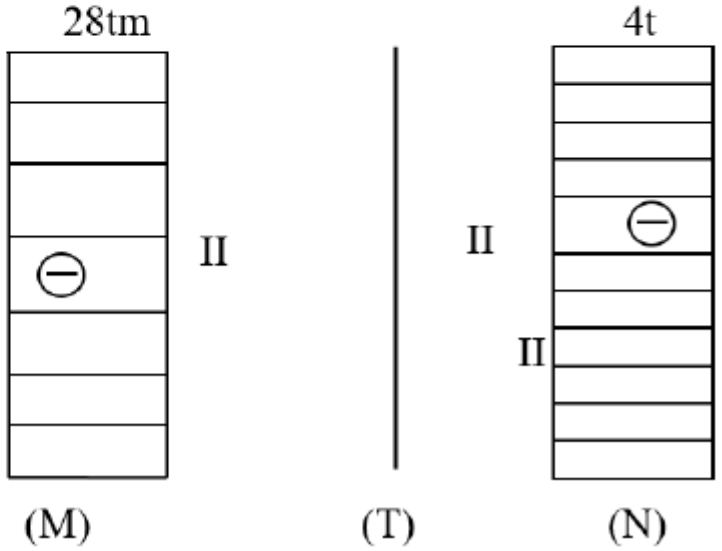
$\Sigma Fx=0 \qquad T_2(x)=0$

$\Sigma Fy=0 \qquad N_2(x)=-4t$

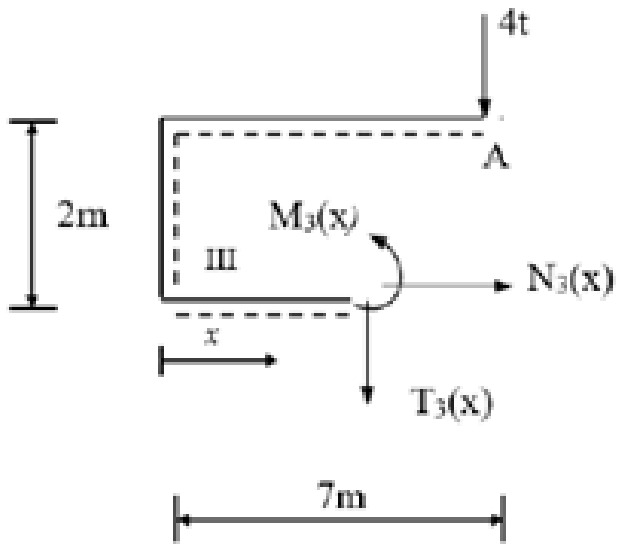
$\Sigma M=0 \qquad M_2(x)+4(7)=0 \quad M_2(x)=-28t$

II bölgesi $0 \le x \le 2m$





III bölgesi $0 \le x \le 3m$



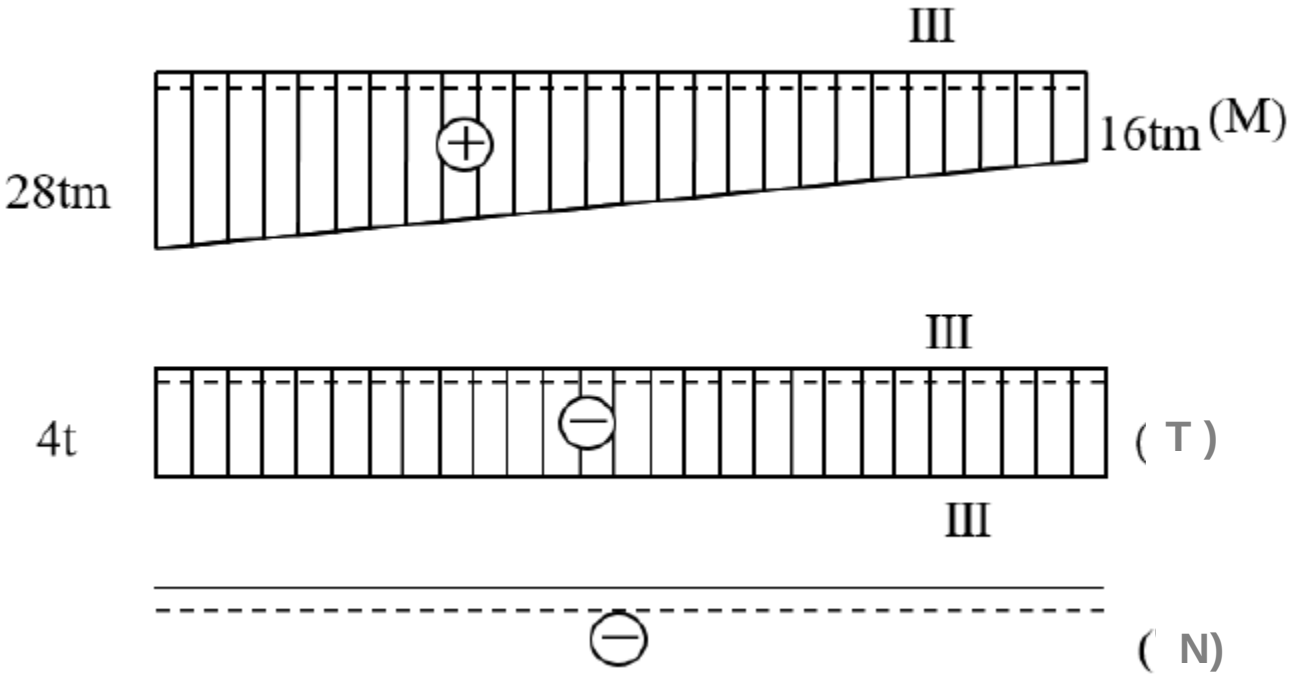
$\Sigma Fx=0$ $N_3(x)=$ 0

$\Sigma Fy=0$ $T_3(x)=-4t$

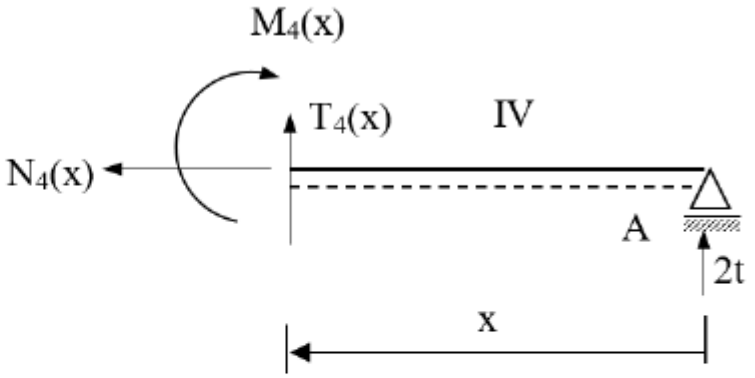
$\Sigma M=0$ $M_3(x) - 4\,(7-x)=0$

$M_3(x)=28-4x$ x=0 M=28 tm

x=3 m M=16 tm


III
28tm
16tm (M)
III
4t
(T)
III
(N)

IV bölgesi $0 \le x \le 8m$

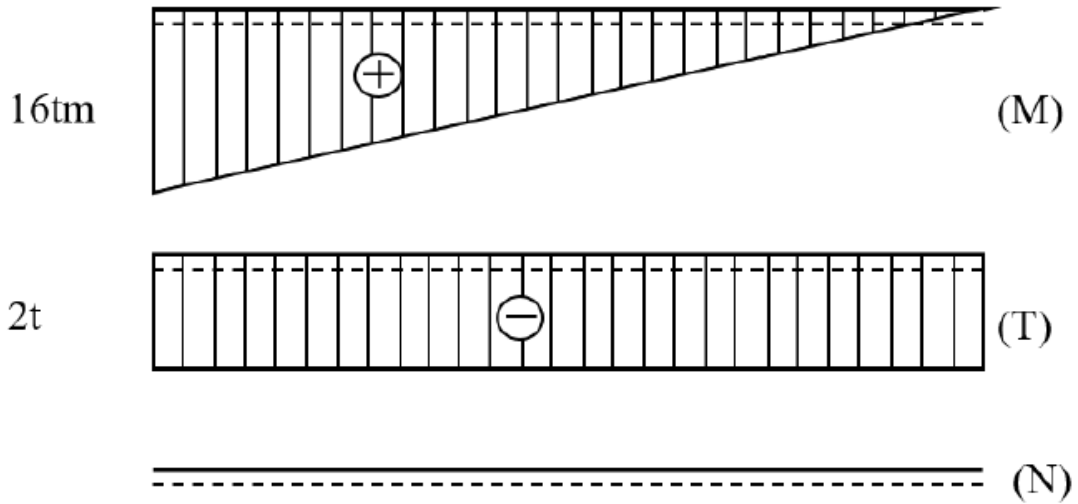


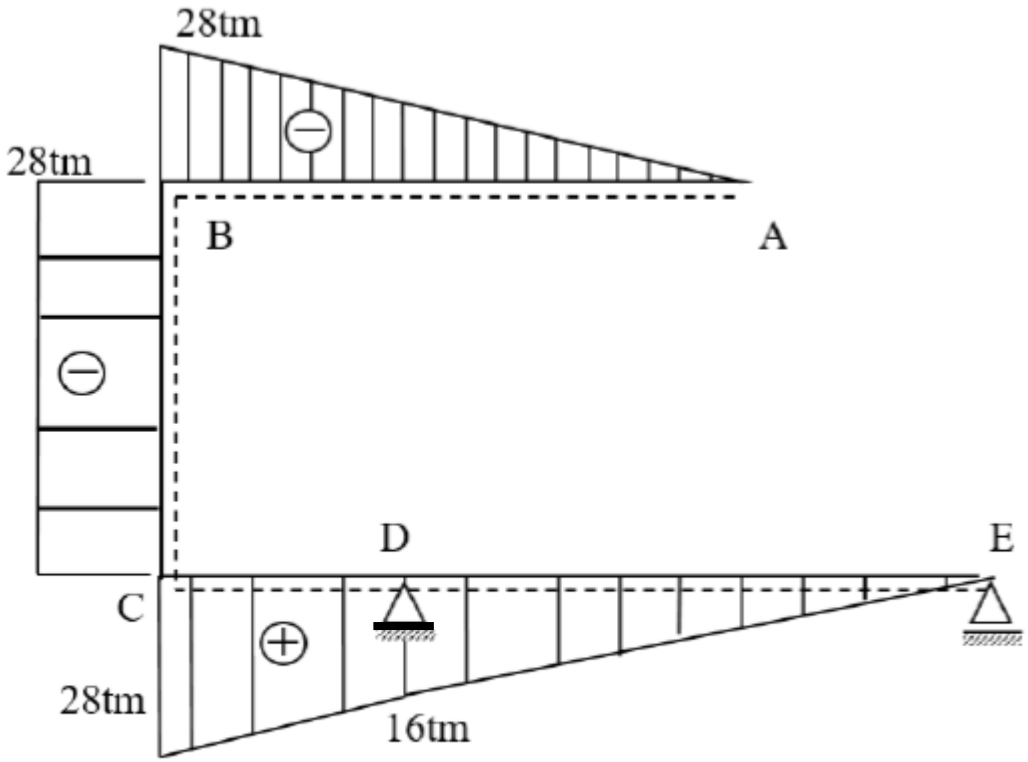
$\Sigma Fx=0$ $N_4(x)=0$

$\Sigma Fy=0$ $T_4(x)= -2t$

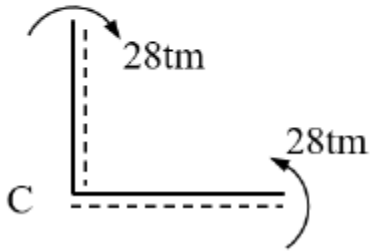
$\Sigma M=0$ $M_4(x)-2\,(x)=0$ $M_4(x)=2x$

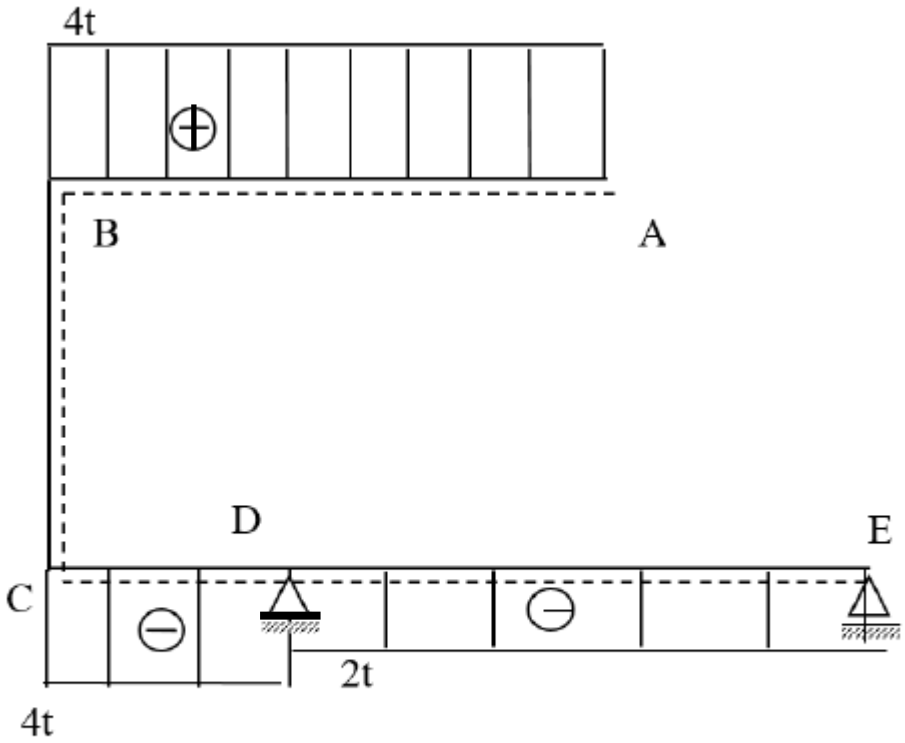
$M_4(x=0)=0$ $M_4\,(x=8)=16$ tm



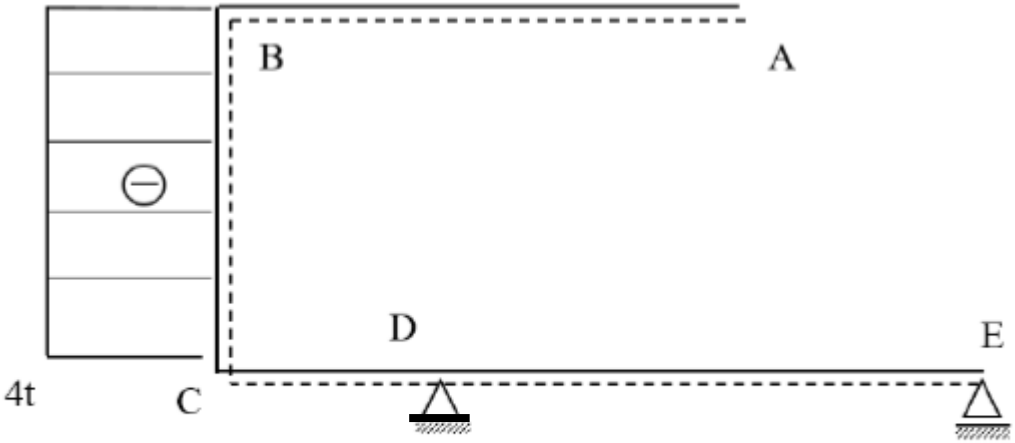


Denge Şartı: Diyagramlar çizildikten sonra düğüm noktalarında moment denge şartı kontrol edilmelidir. Örneğin C noktasında $\Sigma M_C=0$ denge şartı sağlanmıştır.



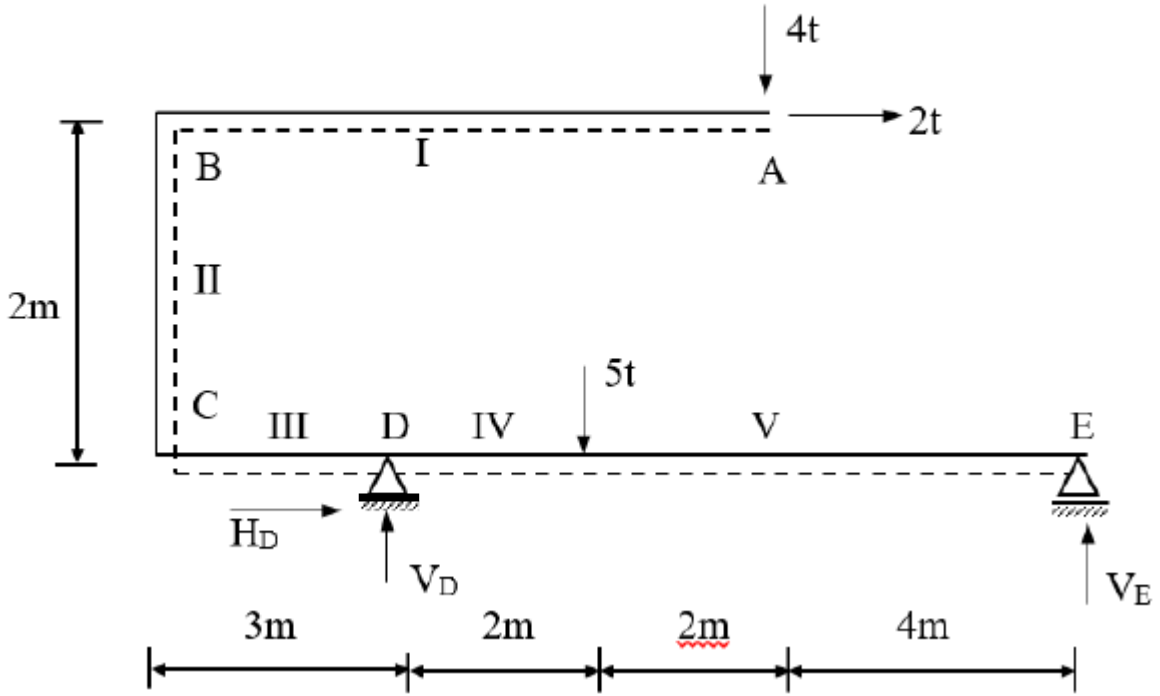


(T)



(N)

PROBLEM 4.5 Şekildeki sistemin M, N, T diyagramlarını çiziniz.

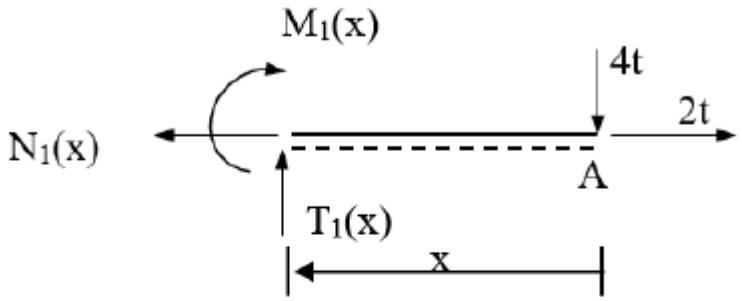


$\Sigma F_x=0 \quad H_D+2=0 \Rightarrow H_D=-2t$

$\Sigma M_D=0 \quad 5.2+4.4+2.2-8.V_E=0 \Rightarrow V_E=3.75t$

$\Sigma F_y=0 \quad V_D+V_E-5-4=0 \Rightarrow V_D+3.75-9=0 \Rightarrow V_D=5.25t$

I bölgesi $0 \le x \le 7m$



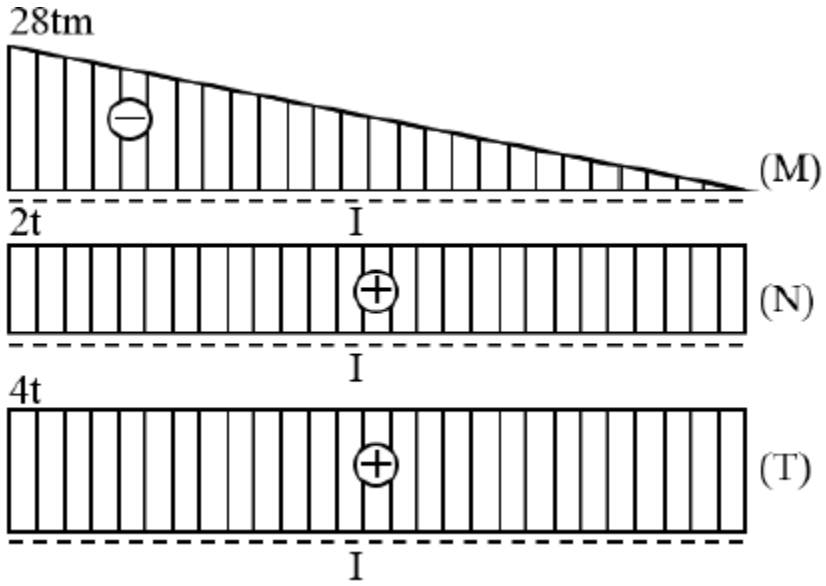
$\Sigma Fx=0$ $\quad -N_1(x)+2=0$ $\quad N_1(x)=2t$

$\Sigma Fy=0$ $\quad T_1(x)-4=0$ $\quad T_1(x)=4t$

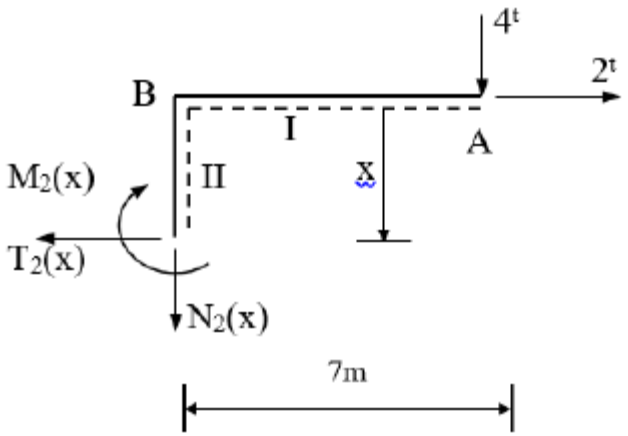
$\Sigma M=0$ $\quad M_1(x)+(4)x=0$ $\quad M_1(x)=-4x$

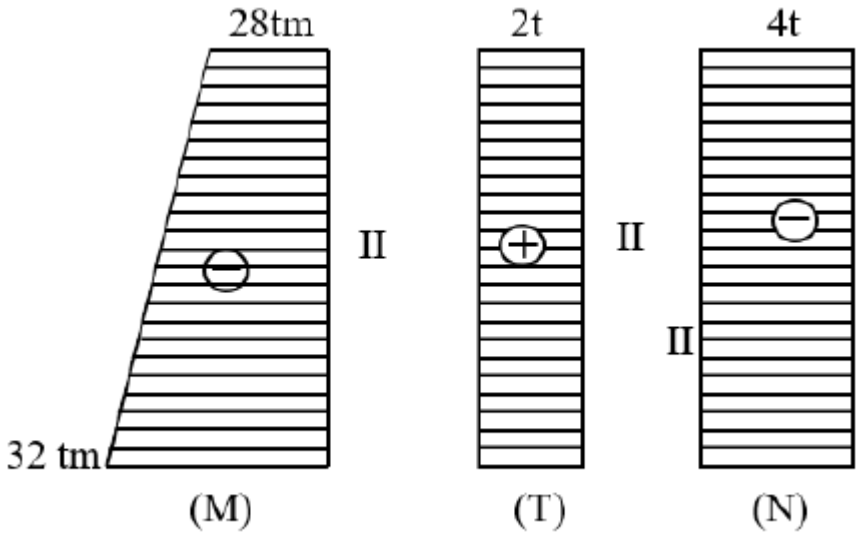
$x=0$ için $\quad M_1(x)=0$

$x=7$ için $\quad M_1(x)=-4(7)=-28t$



II bölgesi $0 \le x \le 2m$





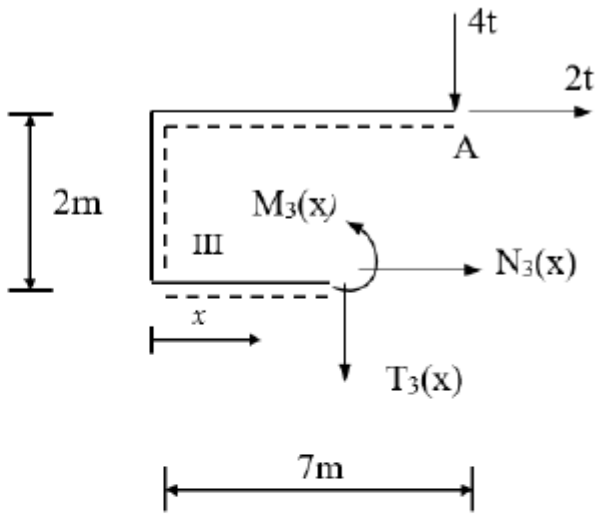
$\Sigma Fx=0$ $\quad T_2(x)=2t$

$\Sigma Fy=0$ $\quad N_2(x)=-4t$

$\Sigma M=0$ $\quad M_2(x)+4(7)+2(x)=0 \quad M_2(x)=-28-2x$

$M_2(x=0)=-28tm, \quad M_2(x=2)=-32tm$

III bölgesi $0 \le x \le 3m$



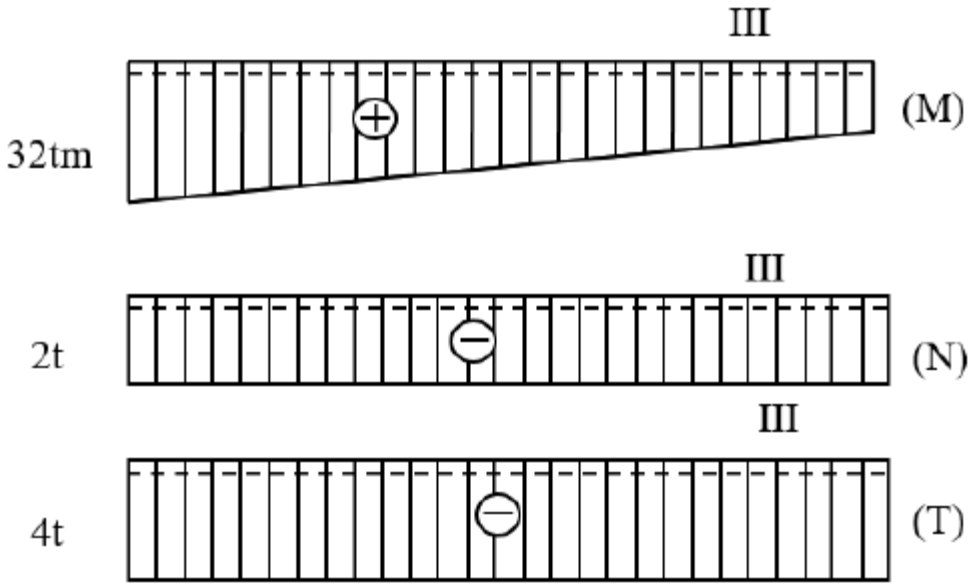
$\Sigma Fx=0$ $N_3(x)=-2t$

$\Sigma Fy=0$ $T_3(x)=-4t$

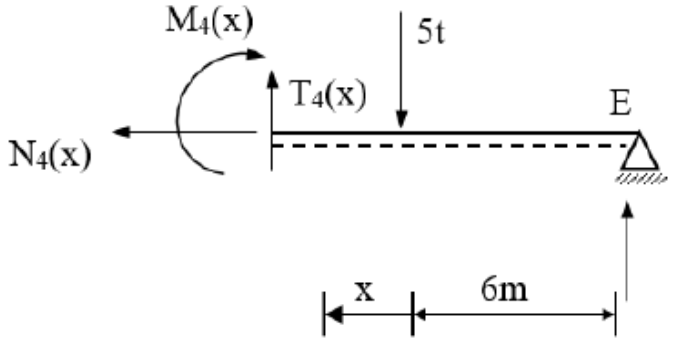
$\Sigma M=0$ $M_3(x) - 4\,(7-x) - 2\,(2)=0$

$M_3(x)=32-4x$ $x=0$ $M=32$ tm

$x=3$ m $M=20$ tm



IV.bölge $0 \le x \le 2m$



$\Sigma Fx=0$ $N_4(x)=0$

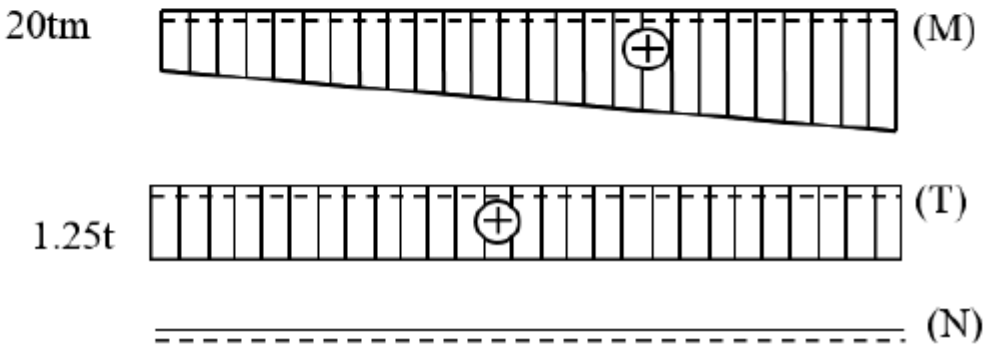
$\Sigma Fy=0$ $T_4(x)=5-3.75=1.25t$

$\Sigma M=0$ $M_4(x)+5(x)-3.75(6+x)=0$

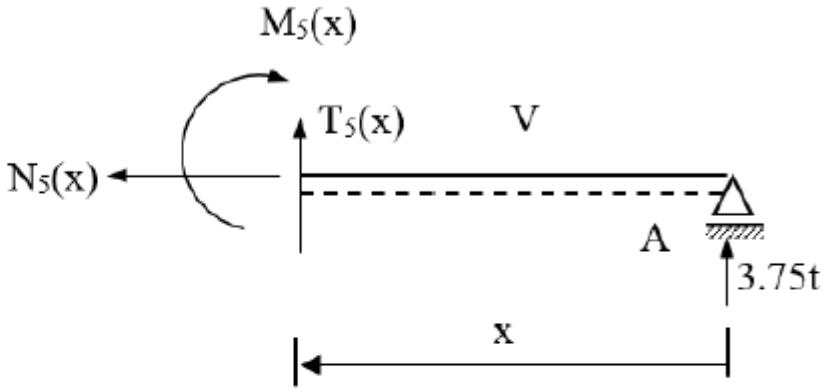
$M_4(x)=-1.25(x)+22.5$

$x=0$ için $M_4(x=0)=22.5$ tm

$x=2$ için $M_4(x=2)=20$ tm



V bölgesi $0 \le x \le 6m$

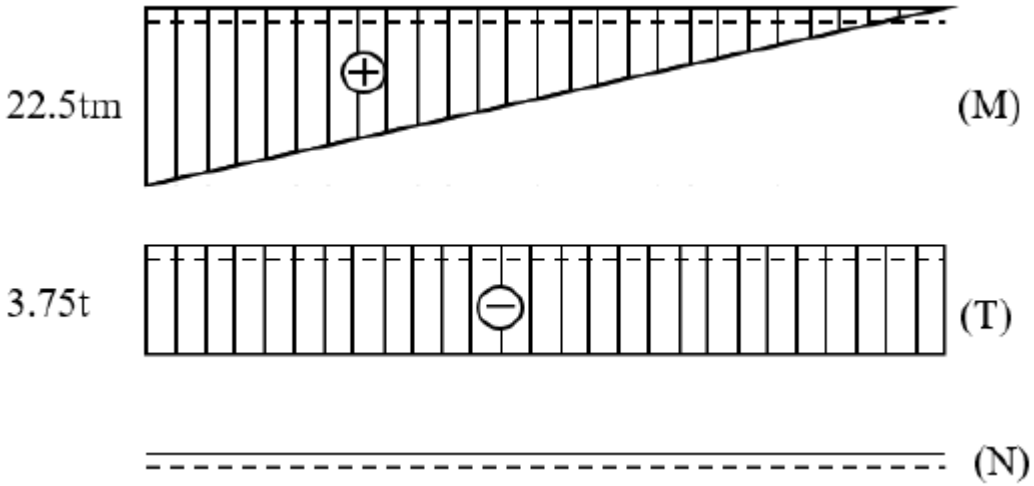


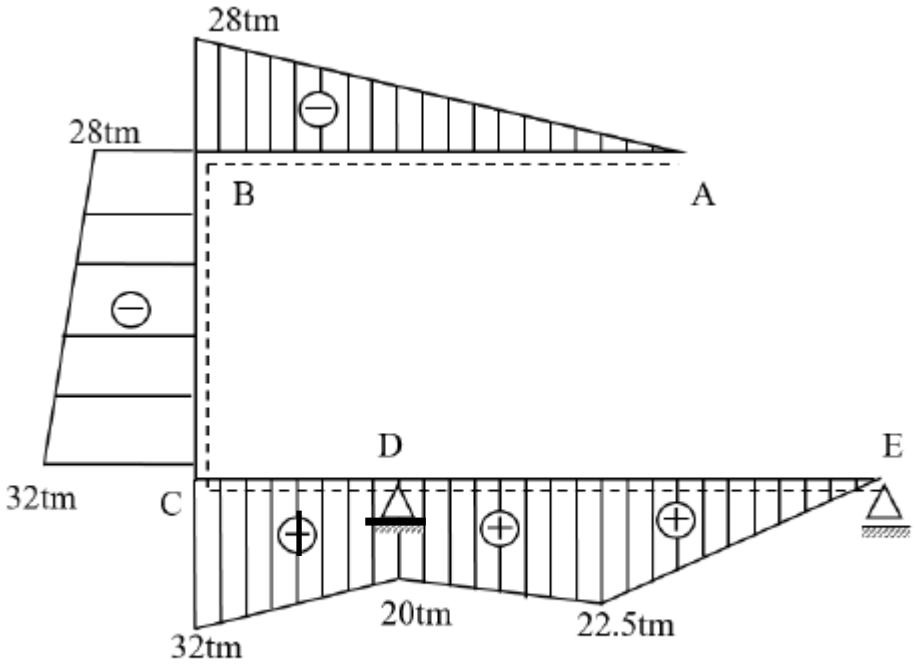
$\Sigma Fx=0$ $N_5(x)=0$

$\Sigma Fy=0$ $T_5(x)= -3.75t$

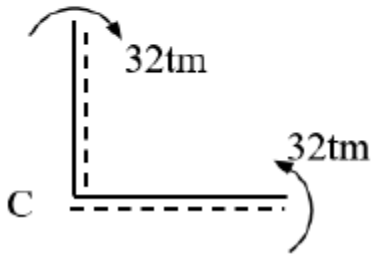
$\Sigma M=0$ $M_5(x)-3.75\,(x)=0$ $M_5(x)=3.75x$

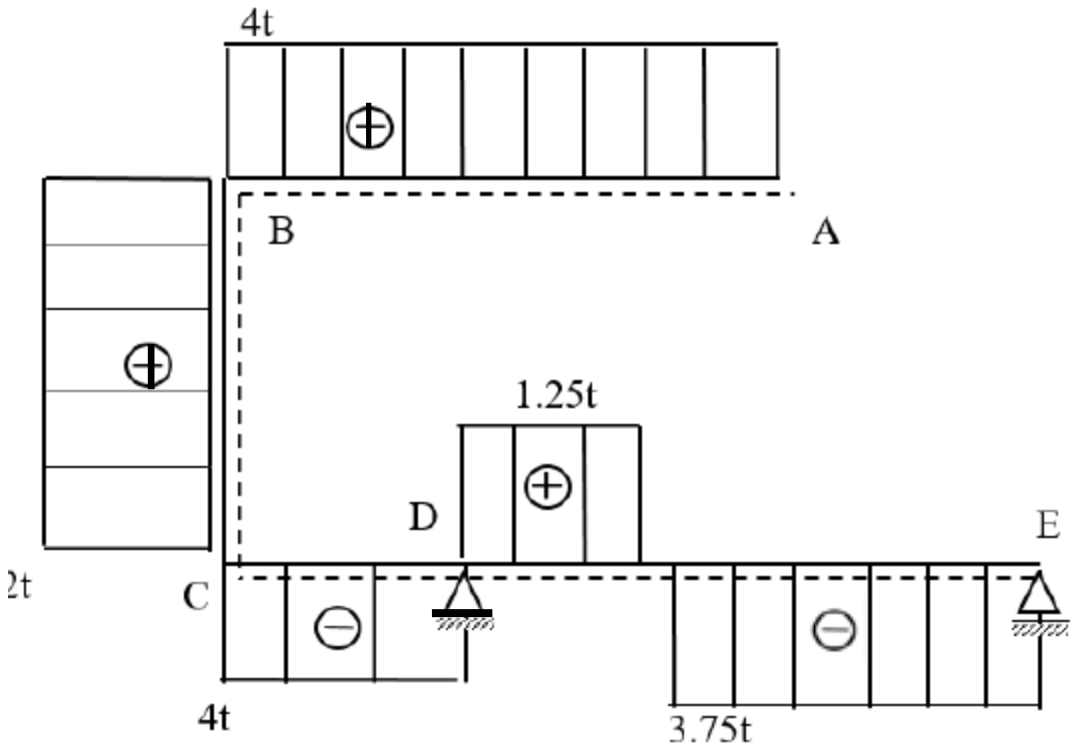
$M_5(x=0)=0$ $M_5(x=6)=22.50$ tm



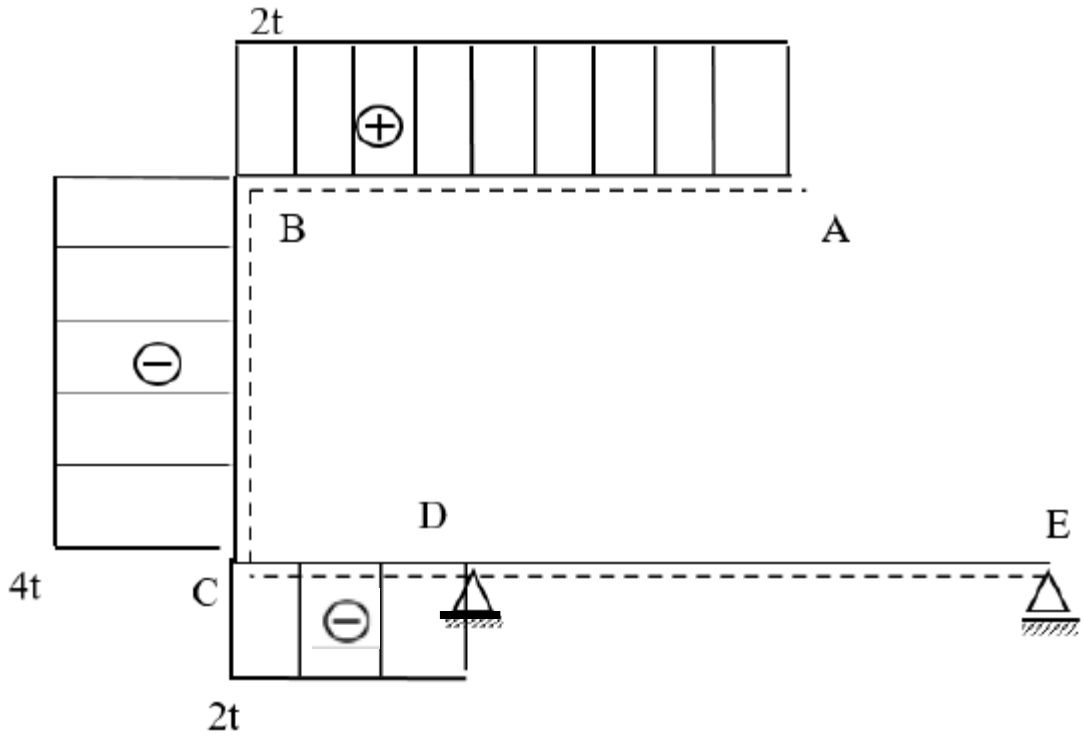


Denge Şartı: Diyagramlar çizildikten sonra düğüm noktalarında moment denge şartı kontrol edilmelidir. Örneğin C noktasında $\Sigma M_C=0$ denge şartı sağlanmıştır.

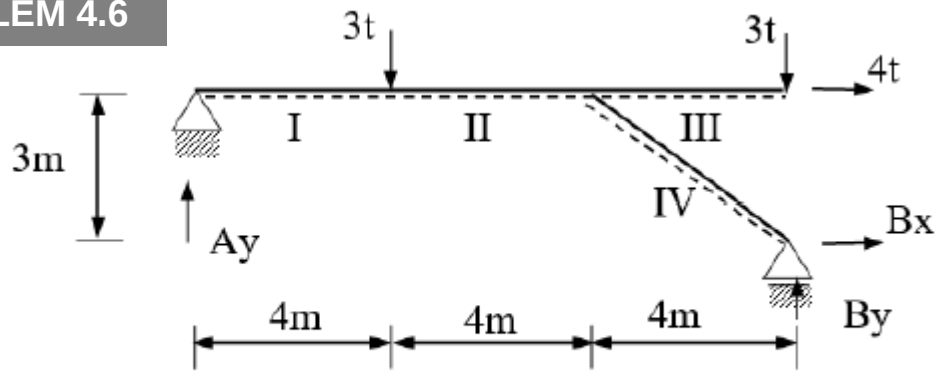




(T)


2t
B
A
D
E
4t
C
2t
(N)

PROBLEM 4.6



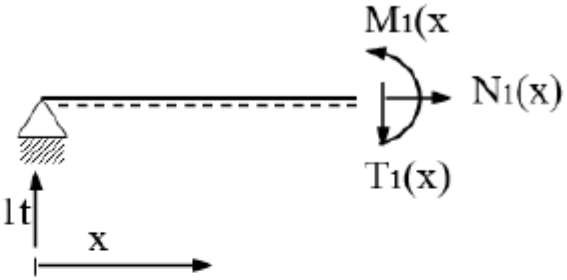
Şekildeki sistemin M, N, T diyagramlarının çiziniz.

$\sum Fx=0$ $\quad Bx-4=0 \Rightarrow Bx=4t$

$\Sigma Fy=0$ $\quad Ay+By-3-3=0 \Rightarrow Ay+By=6t$

$\Sigma M_B=0$ $\quad Ay\ 12-3\ 8+4\ 3=0 \Rightarrow Ay=1t \quad By=5t$

I bölgesi $0 \le x \le 4m$



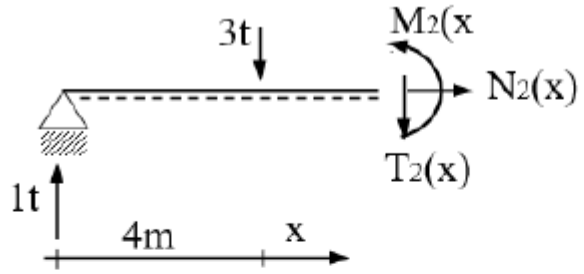
$\Sigma Fx=0 \Rightarrow N_1(x)=0$

$\Sigma Fy=0 \Rightarrow -T_1(x)+1=0 \Rightarrow T_1(x)=1$

$\Sigma M=0 \Rightarrow M_1(x)-1\ x=0 \Rightarrow M_1(x)=x$

$M_1(0)=0 \quad M_1(4)=4$ tm

II bölge $0 \le x \le 4m$



$\Sigma Fx=0 \qquad N_2(x)=0$

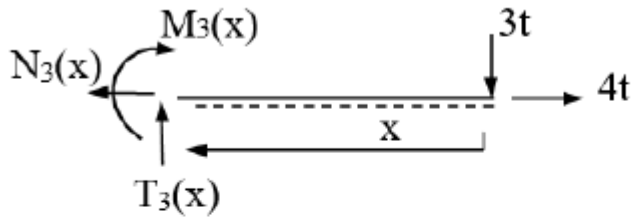
$\Sigma Fy=0 \qquad 1-3-T_2(x)=0$

$T_2(x)= -2t$

$\Sigma M=0 \qquad M_2(x)+3x-1\ (4+x)=0 \qquad M_2(x)= -2x+4$

$M_2(0)=4tm \qquad M_2(4)= -4tm$

III bölge $0 \le x \le 4m$



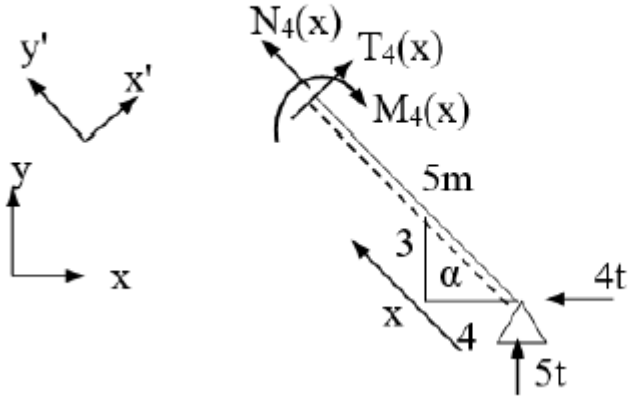
$\Sigma Fx=0 \qquad N_3(x)=4t$

$\Sigma Fy=0 \qquad T_3(x)=3t$

$\Sigma M=0 \qquad M_3(x)= -3x$

$M_3(0)=0 \qquad M_3(4)= -12tm$

IV bölge $0 \le x \le 5m$



$\cos\alpha=4/5=0.8$
$\sin\alpha=3/5=0.6$

$\Sigma Fy'=0$ $\quad N_4(x)+5\ \sin\alpha+4\ \cos\alpha=0$

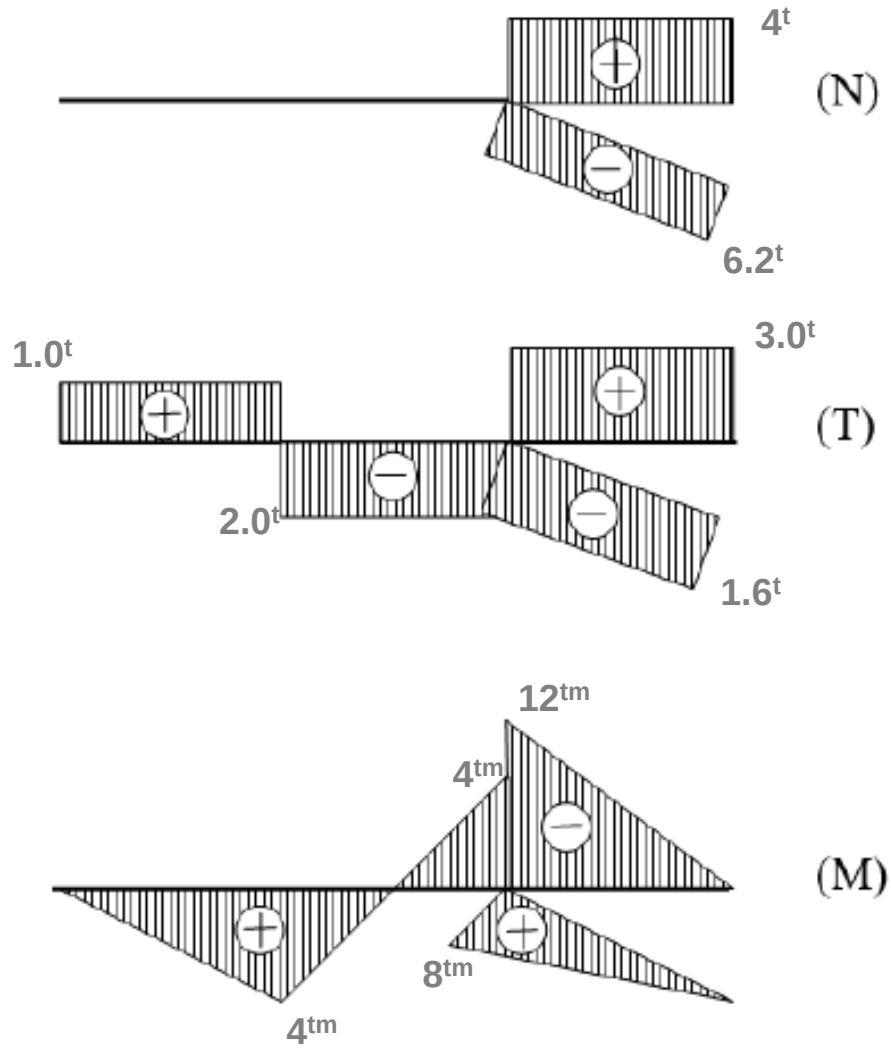
$N_4(x)= -6{,}2t$

$\Sigma Fx'=0$ $\quad T_4(x)+5\ \cos\alpha -4\ \sin\alpha=0$

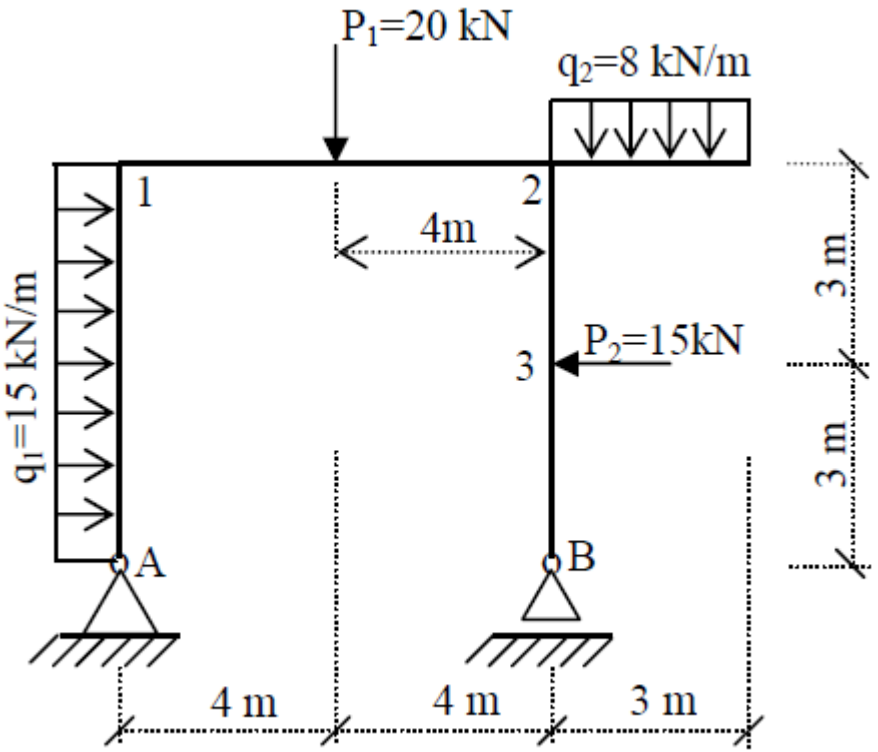
$T_4(x)= -1{,}6t$

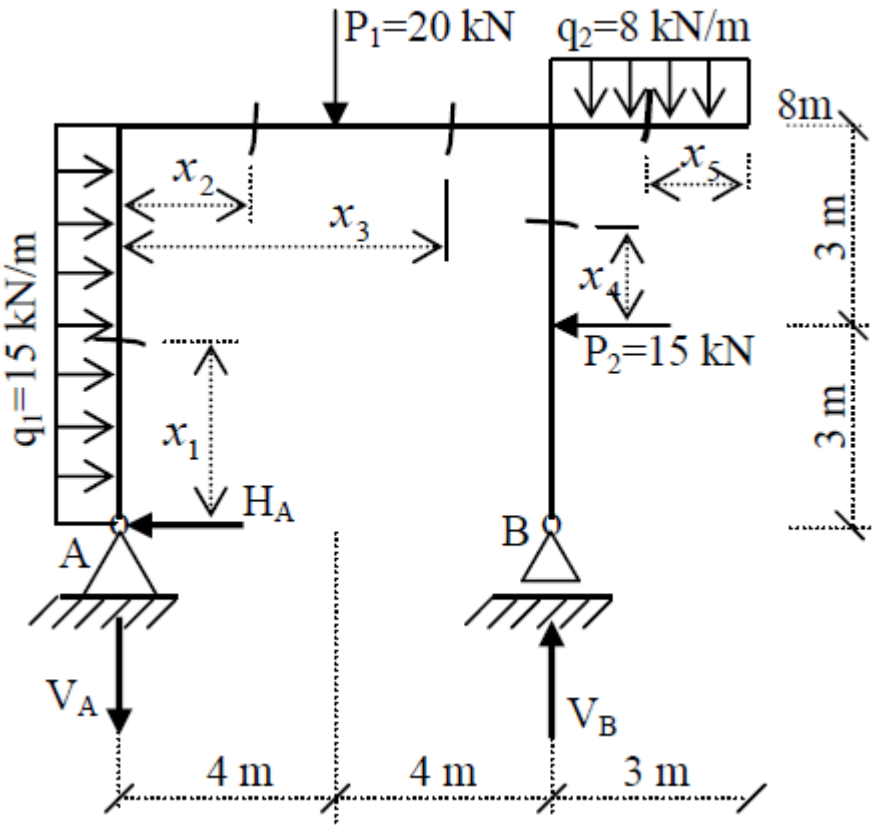
$\Sigma M=0$ $\quad M_4(x)-5\ \cos\alpha\ x+4\ \sin\alpha\ x=0$ $\quad M_4(x)=1.6x$

$M_4(0)=0$ $\quad M_4(5)=8tm$


4t
(N)
6.2t
1.0t
3.0t
(T)
2.0t
1.6t
12tm
4tm
(M)
8tm
4tm

PROBLEM 4.6 Şekildeki çerçevenim M, N. T diyagramlarını çerçeve bölgelerinde kesim yaparak çiziniz (A-1, 1-2, 3-2, B-3)

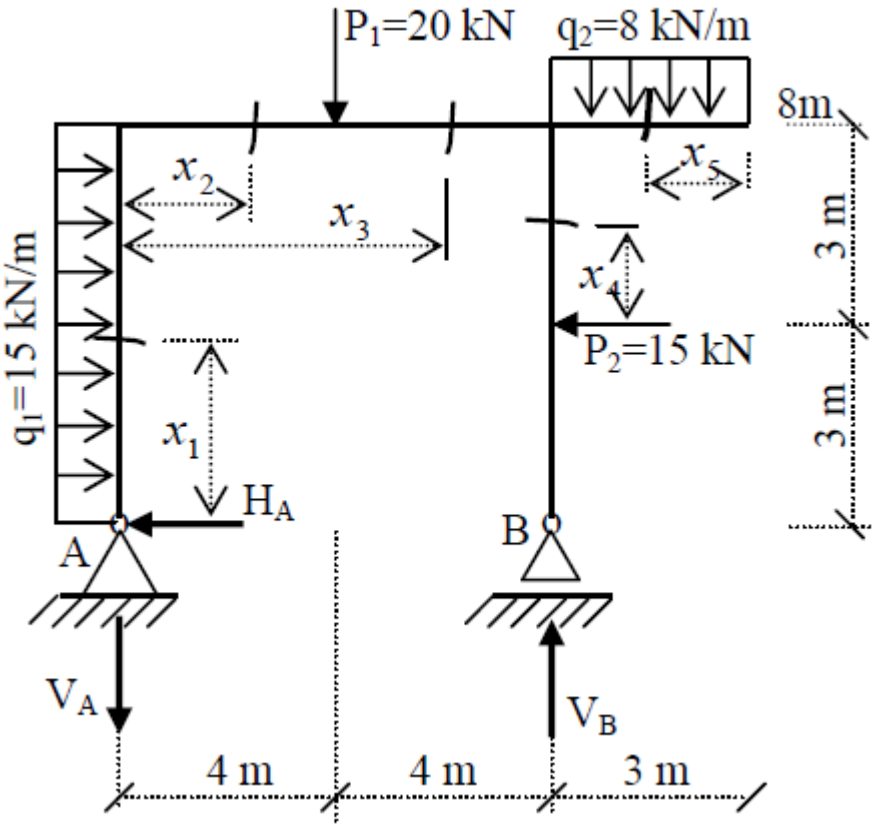




$$\sum X = 0 \Rightarrow q_1.6 - H_A - P_2 = 0 \Rightarrow H_A = 15.6 - 15 = 75\,kN$$

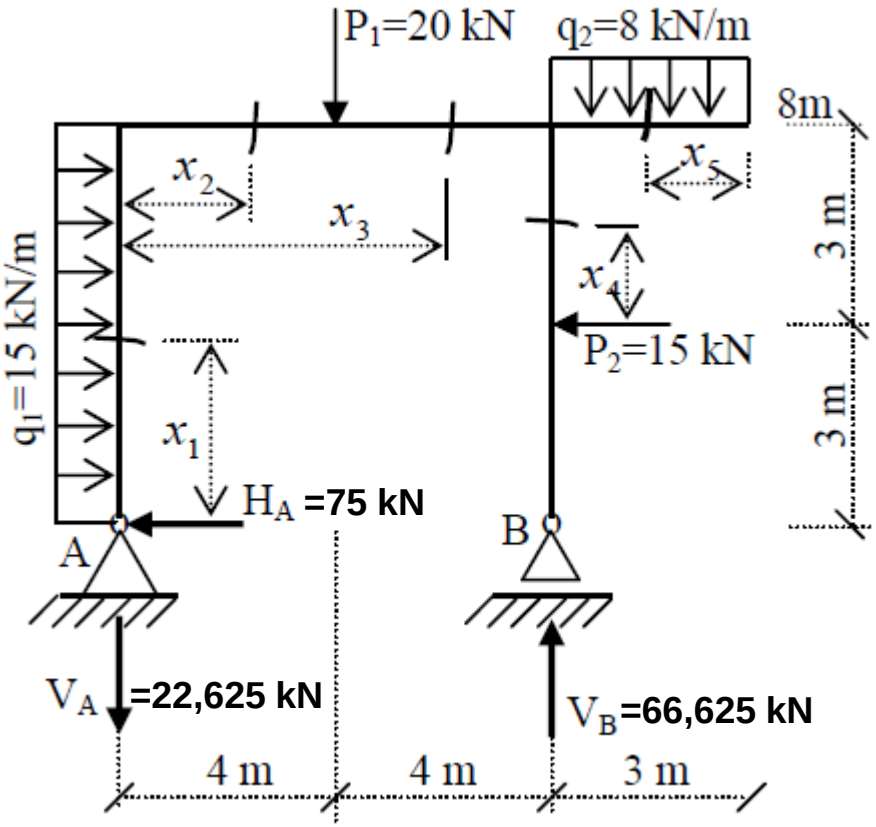
$$\sum M_B = -V_A.8 + q_1.6.3 - P_1.4 + q_2.3.(1{,}5) - P_2.3 = 0 \Rightarrow$$

$$V_A = \frac{15.6.3 - 20.4 + 8.3.(1{,}5) - 15.3}{8} = 22{,}625\,kN$$

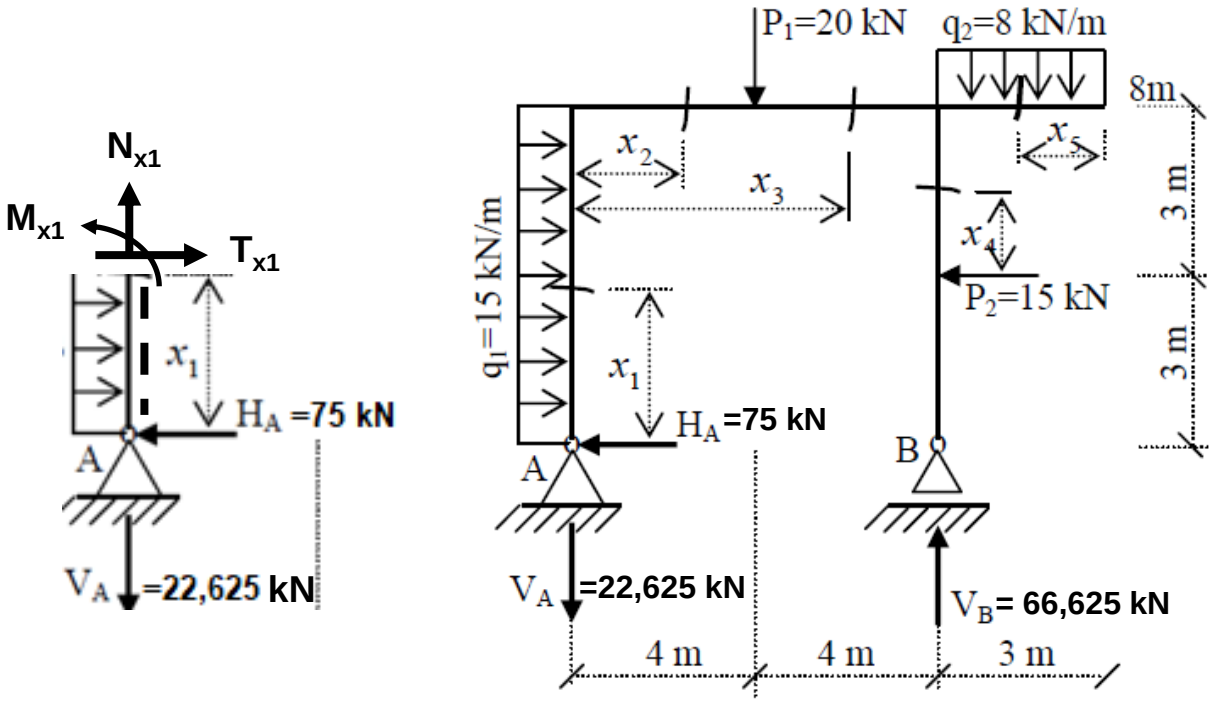


$$\sum M_A = q_2.3.(9{,}5) - P_2.3 - V_B.8 + P_1.4 + q_1.6.3 = 0$$

$$V_B = \frac{8.3.(9{,}5) - 15.3 + 20.4 + 15.6.3}{8} = 66{,}625\,kN$$



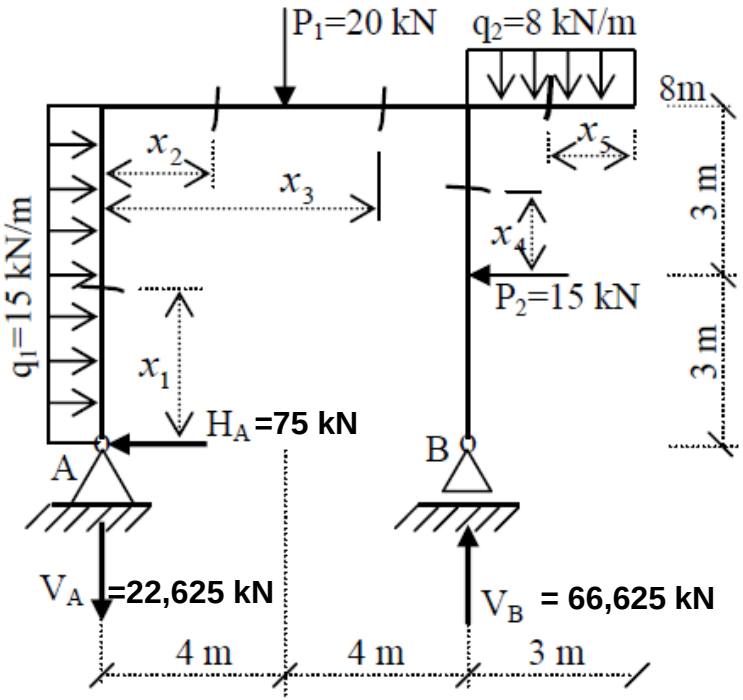
$$\sum Y = -V_A - P_1 - q_2.3 + V_B = 0 \Rightarrow -22{,}625 - 20 - 8.3 + 66{,}625 = 0$$



$$0 \le x_1 \le 6$$

$$M_{x1} = H_A.x_1 - q_1.x_1.\frac{x_1}{2} \;;\; T_{x1} = H_A - q_1.x_1 \;;\; N = V_A = 22{,}625\,kN$$

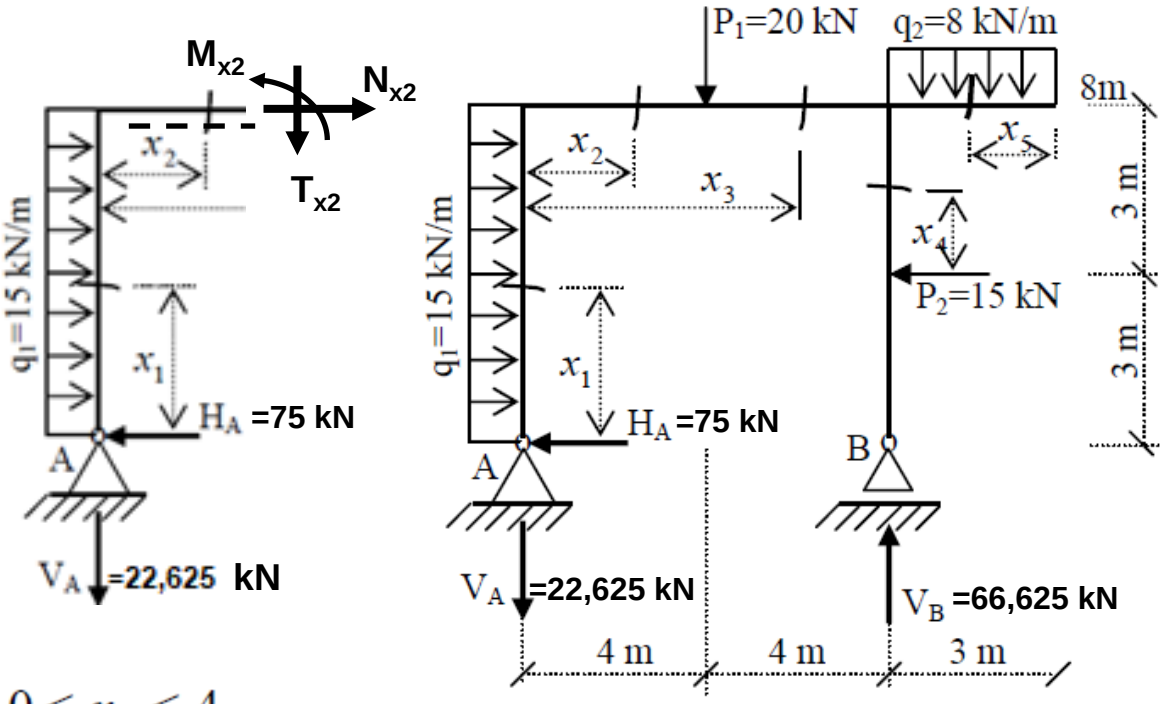
$$x_1 = 0 \;\Rightarrow\; M_{x1} = 0\,, \quad T_{x1} = H_A = 75\,kN.$$



Eğilme momentinin maksimum değer aldığı kesitin yeri:

$$T_{x1} = H_A - q_1.x_1 = 0 \Rightarrow \quad 75 - 15.x_1 = 0 \quad \Rightarrow \quad x_1 = 5\,m. \quad M_{x1}^{\max} = 75.5 - 15.\frac{5^2}{2} = 187{,}5\,kN.m$$

$$x_1 = 6\,m \quad \Rightarrow \; M_{x1} = 75.6 - 15.\frac{6^2}{2} = 180\,kN.m\,, \qquad T_{x1} = 75 - 15.6 = -15\,kN.$$

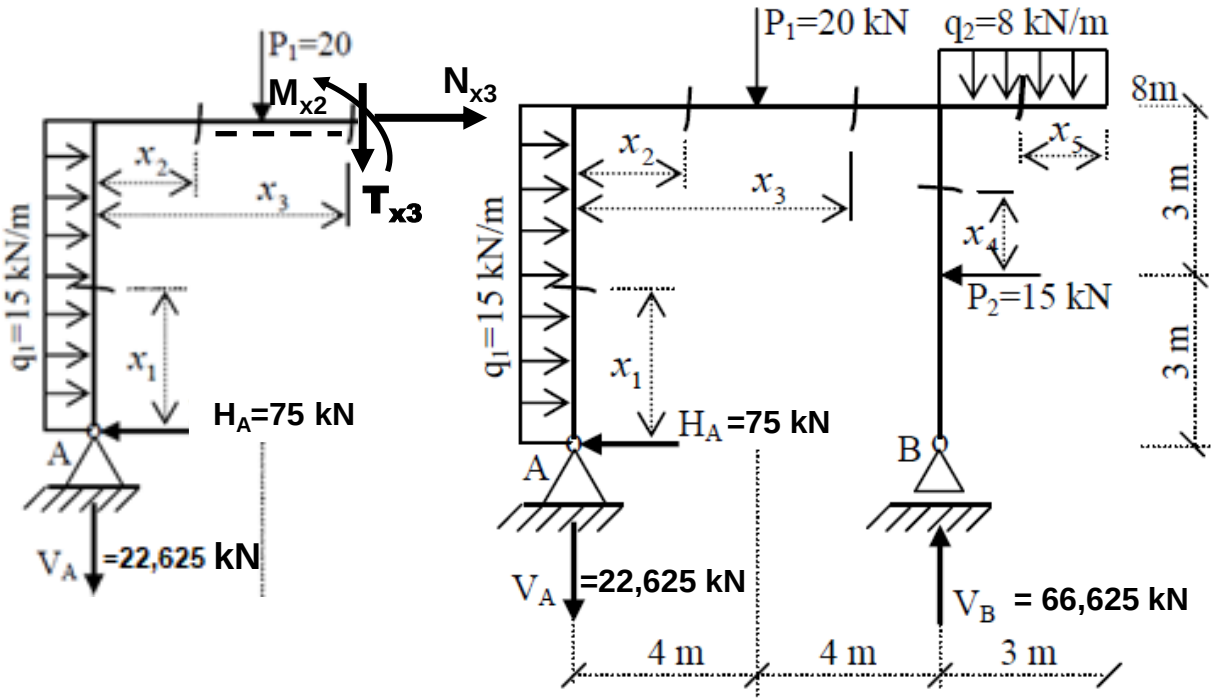


$$0 \le x_2 \le 4$$

$$M_{x2} = H_A.6 - q_1.\frac{6^2}{2} - V_A.x_2 = 75.6 - 15.\frac{6^2}{2} - (22{,}625).x_2 = 180 - (22{,}625).x_2$$

$$T = -V_A = -22{,}625\,kN\,; \quad N = -q_1.6 + H_A = -15.6 + 75 = -15\,kN.$$

$$x_2 = 0 \Rightarrow \quad M_{x2} = 180\,kN.m\,, \quad x_2 = 4\,m \Rightarrow \quad M_{x2} = 180 - (22{,}625).4 = 89{,}5\,kN.m$$

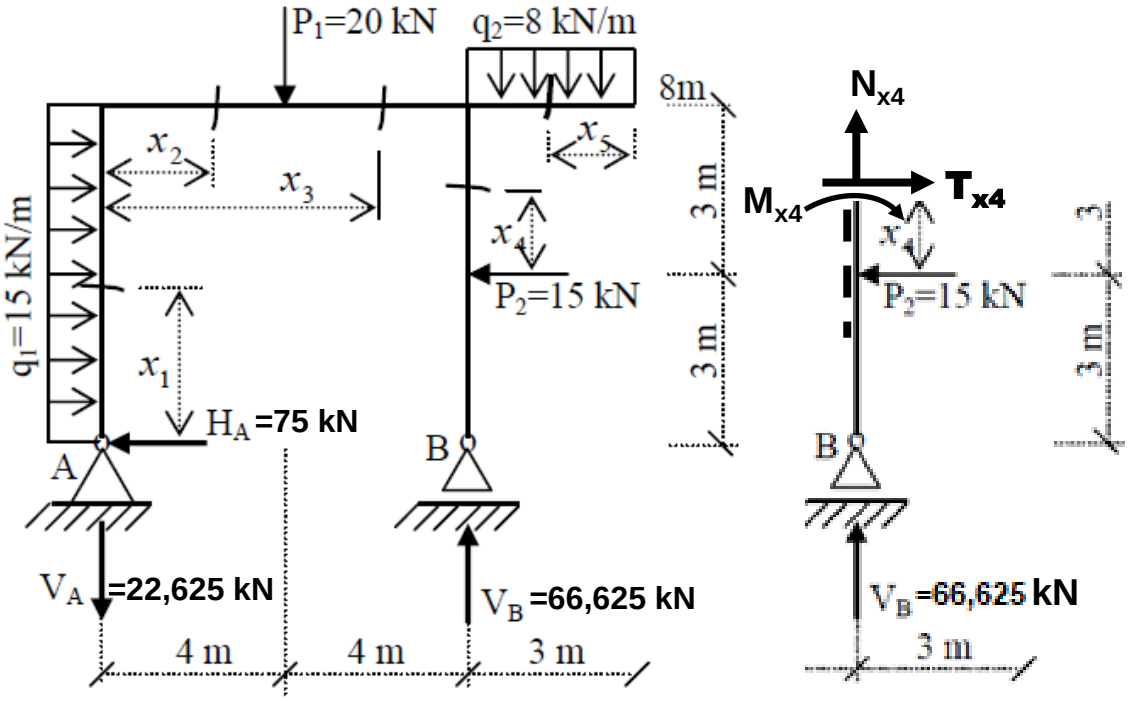


180 kNm

$$4 \le x_3 \le 8 \quad M_{x3} = q_1 6 \frac{6}{2} - H_A 6 - V_A x_3 - P_1(x_3 - 4) = 180 - 22{,}625x_3 - 20(x_3 - 4)$$

$$M_{x3} = 180 - V_A.x_3 - P_1.(x_3 - 4)\,;\; T = -V_A - P_1 = -22{,}625 - 20 = -42{,}625\,kN\,;\; N = -15\,kN.$$

$$x_3 = 4m \Rightarrow \; M_{x3} = 89{,}5\,kN.m\,,\; x_3 = 8\,m \Rightarrow \; M_{x3} = 180 - (22{,}625).8 - 20.4 = -81\,kN.m.$$



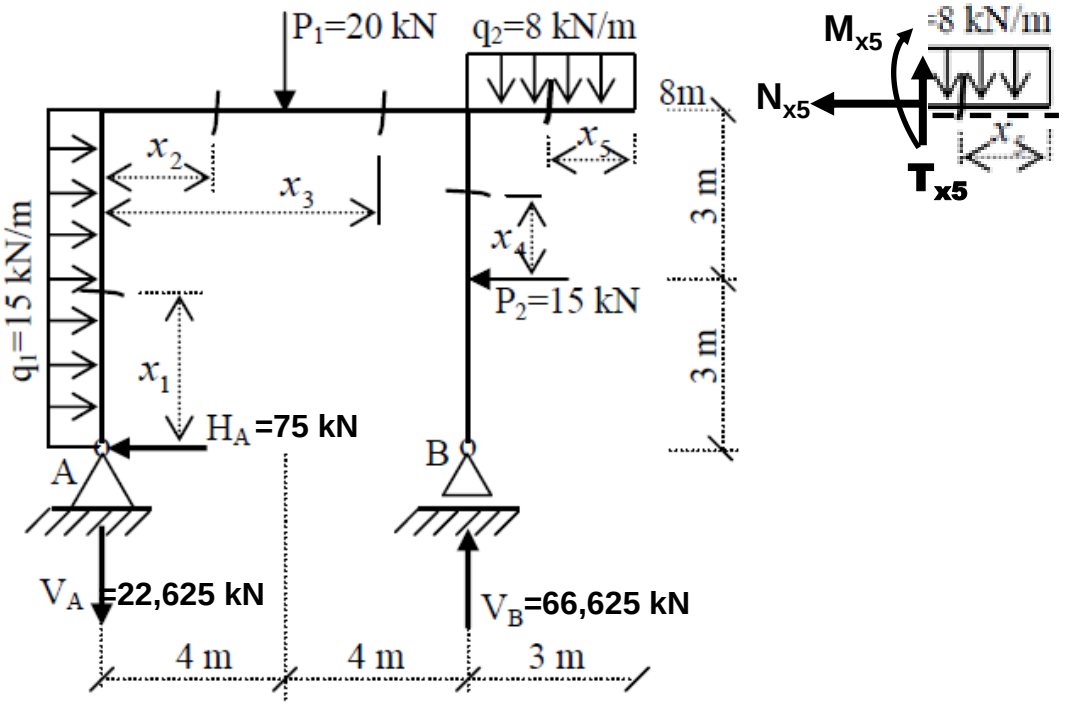
B-3 bölgesinde,

$M = 0; \quad T = 0; \quad N = -V_B = -66{,}625\,kN$ ' dur.

$0 \le x_4 \le 3$

$M_{x4} = -P_2.x_4\,; \quad T = P_2 = 15\,kN\,; \quad N = -V_B = -66{,}625\,kN$

$x_4 = 0 \Rightarrow \quad M_{x4} = 0\,, \quad x_4 = 3\,m \Rightarrow \quad M_{x4} = -15.3 = -45\,kN.m$

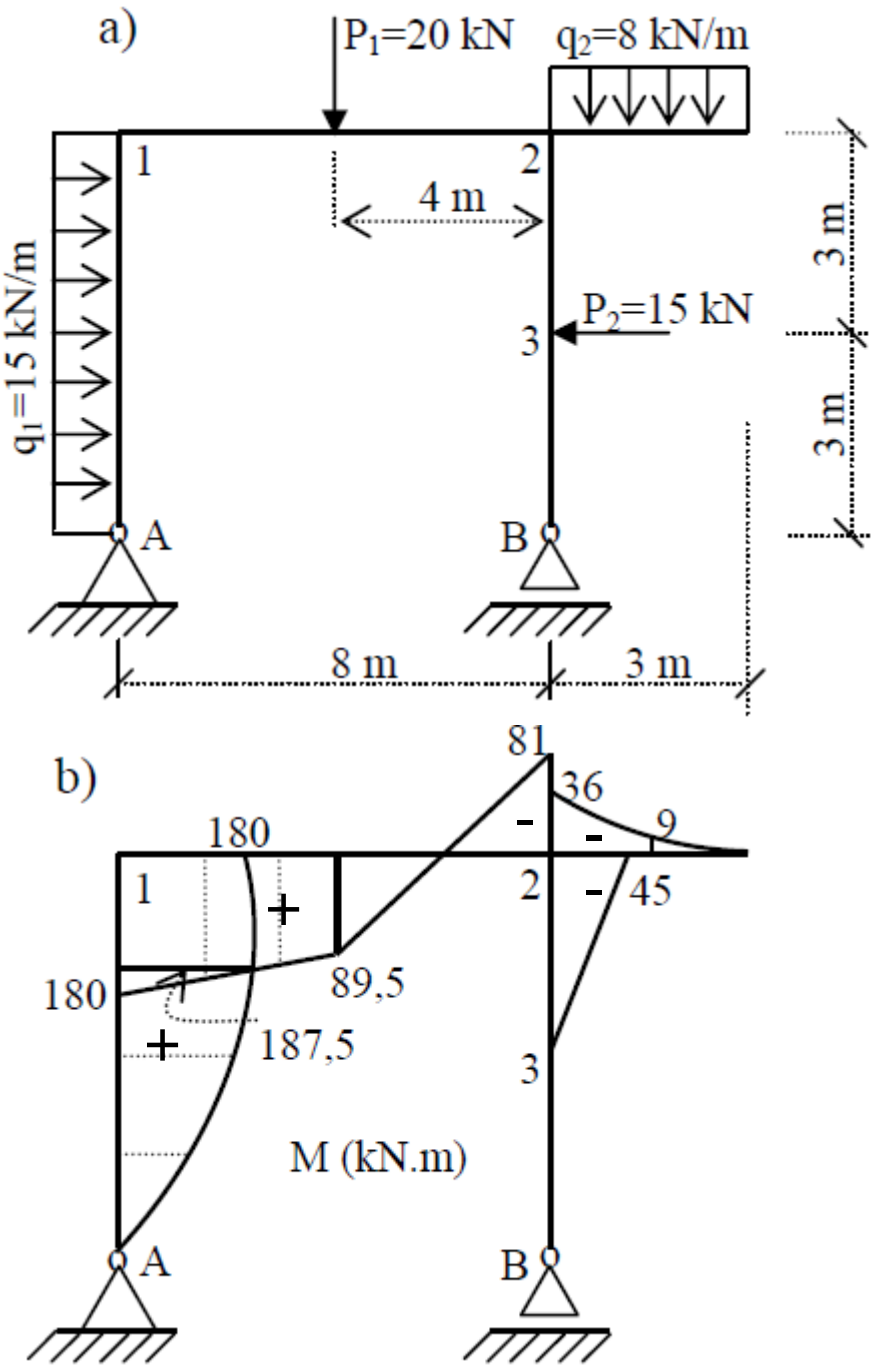


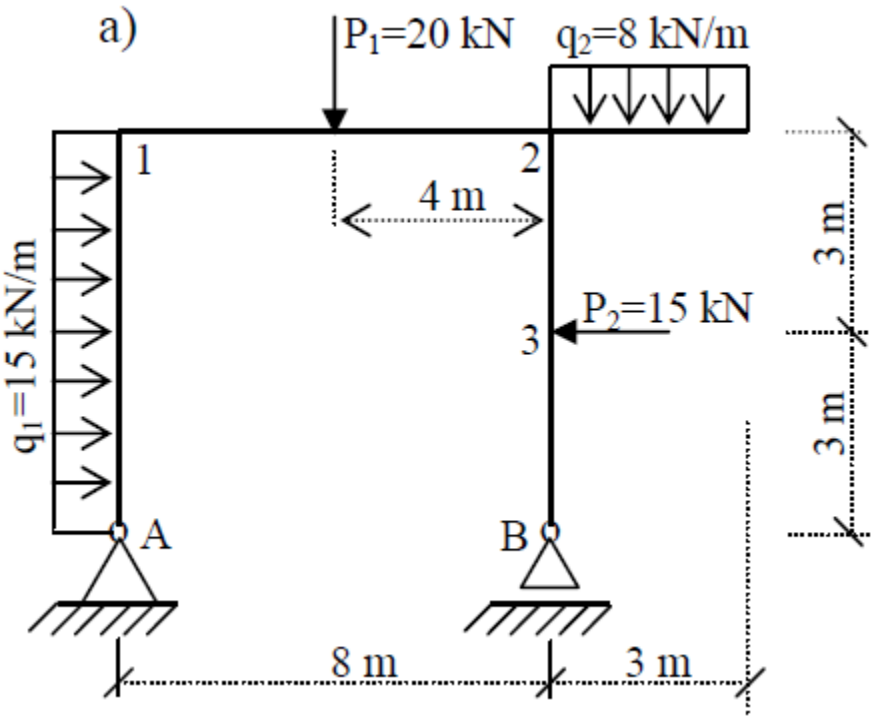
$$0 \le x_5 \le 3$$

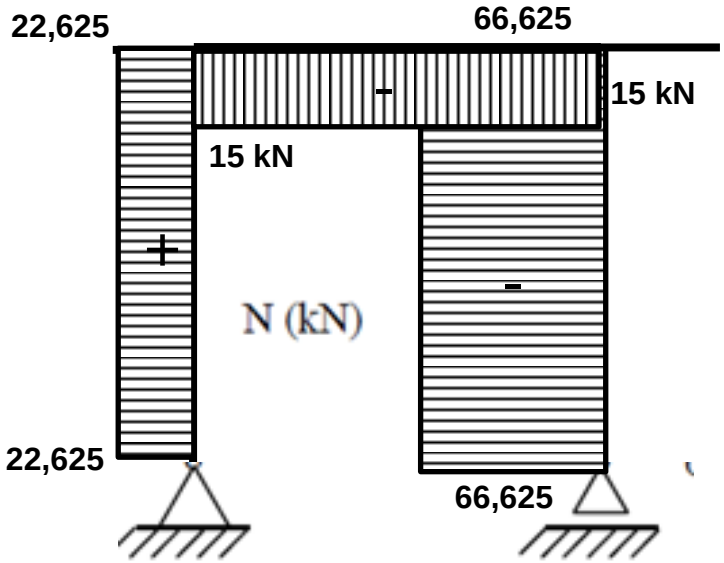
$$M_{x5} = -q_2.x_5.\frac{x_5}{2} = -q_2.\frac{x_5^2}{2}\,; \quad T_{x5} = q_2.x_5\,; \quad N = 0$$

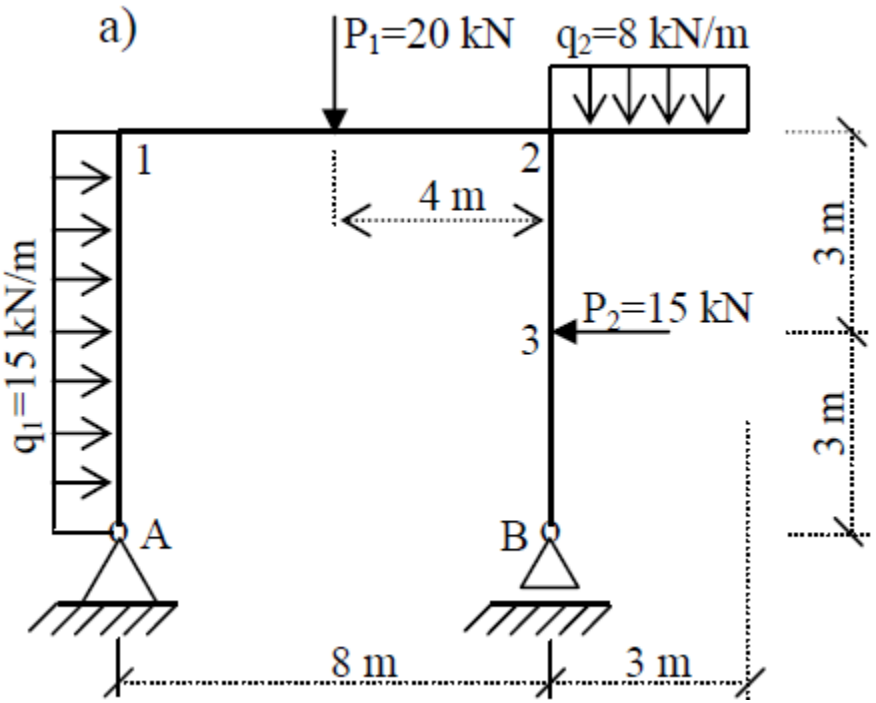
$$x_5 = 0 \Rightarrow \quad M_{x5} = 0\,; \quad T_{x5} = 0\,; \quad x_5 = 1{,}5\,m \Rightarrow \quad M_{x5} = -8.\frac{(1{,}5)^2}{2} = -9\,kN.m$$

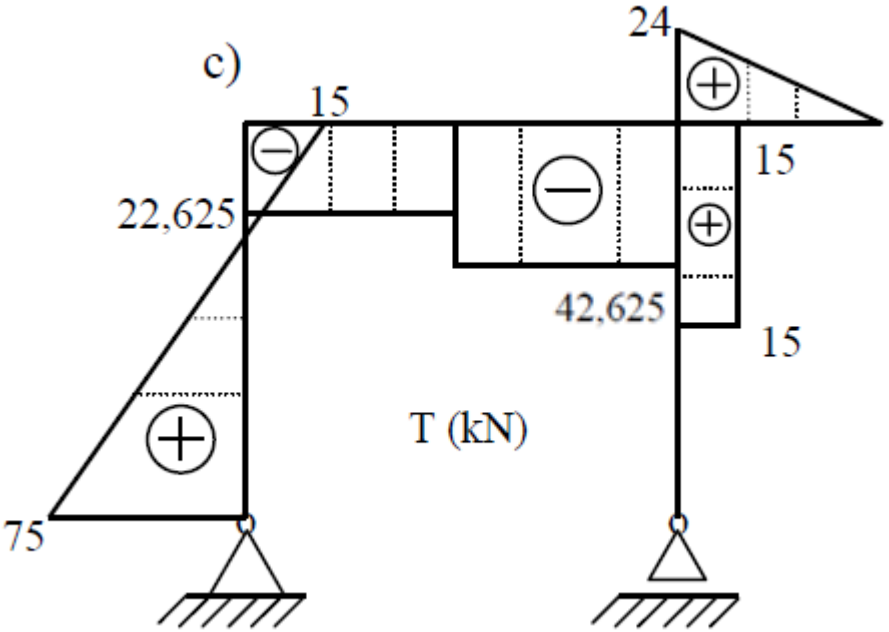
$$x_5 = 3\,m \Rightarrow \quad M_{x5} = -8.\frac{(3)^2}{2} = -36\,kN.m\,; \quad T_{x5} = 8.3 = 24\,kN$$

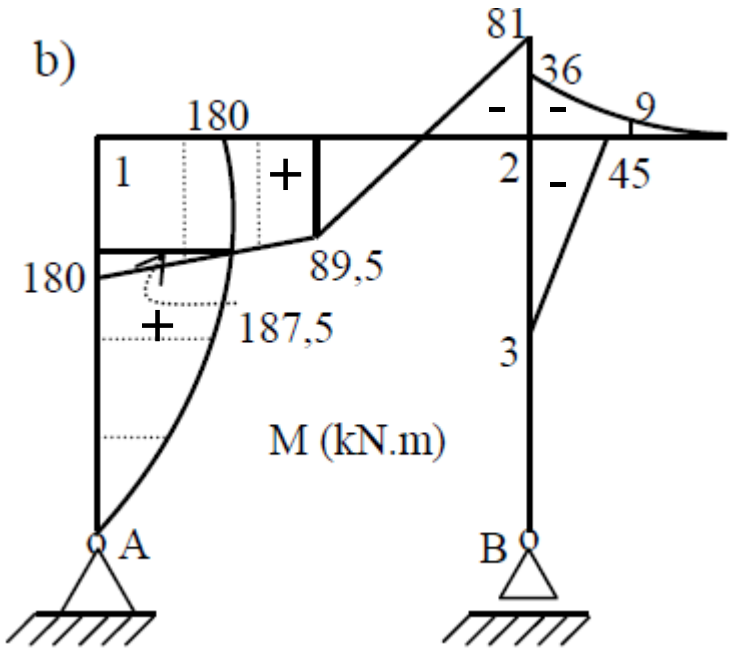

a)
P1=20 kN
q2=8 kN/m
1
2
4 m
3 m
3 m
P2=15 kN
3
q1=15 kN/m
A
B
8 m
3 m
b)
81
36
9
180
1
2
45
180
89,5
187,5
3
M (kN.m)
A
B


a)
P1=20 kN
q2=8 kN/m
1
2
4 m
3 m
P2=15 kN
3
q1=15 kN/m
3 m
A
B
8 m
3 m


22,625
66,625
15 kN
15 kN
N (kN)
22,625
66,625


a)
P1=20 kN
q2=8 kN/m
1
2
4 m
3 m
P2=15 kN
3
q1=15 kN/m
3 m
A
B
8 m
3 m


c)
24
15
15
22,625
42,625
15
T (kN)
75

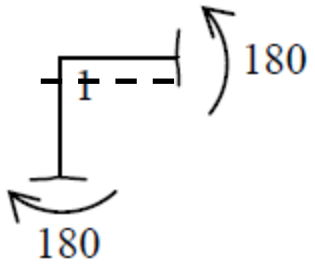


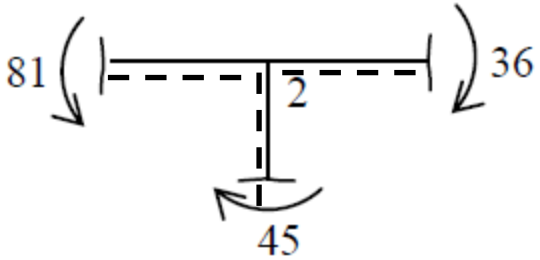
Çerçevenin genel denge kontrolü :

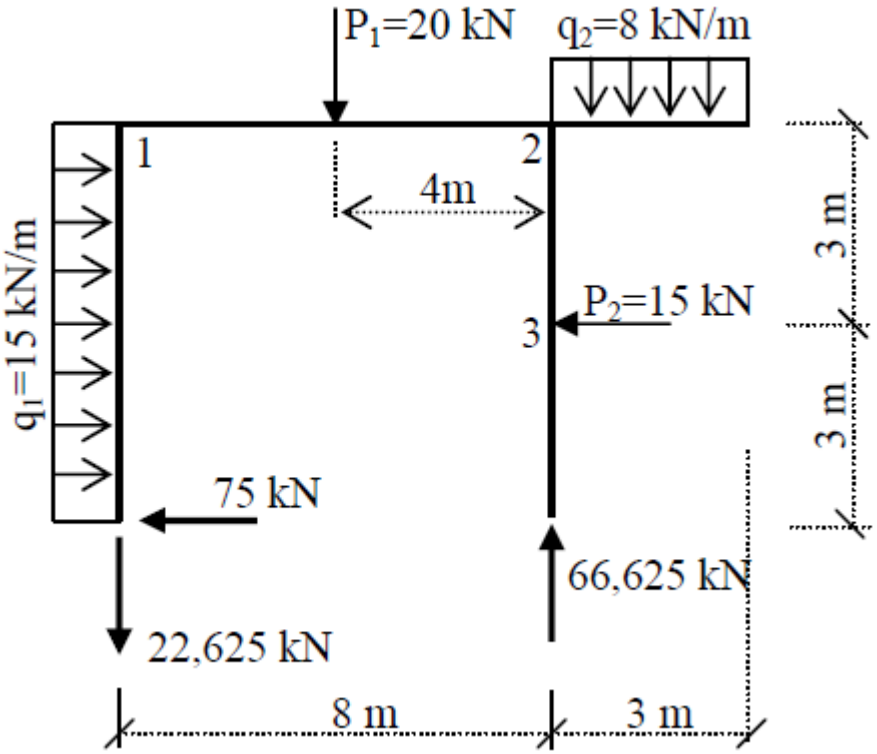
M diyagramından

$$\sum M_1 = 180 - 180 = 0$$

$$\sum M_2 = 36 + 45 - 81 = 0$$





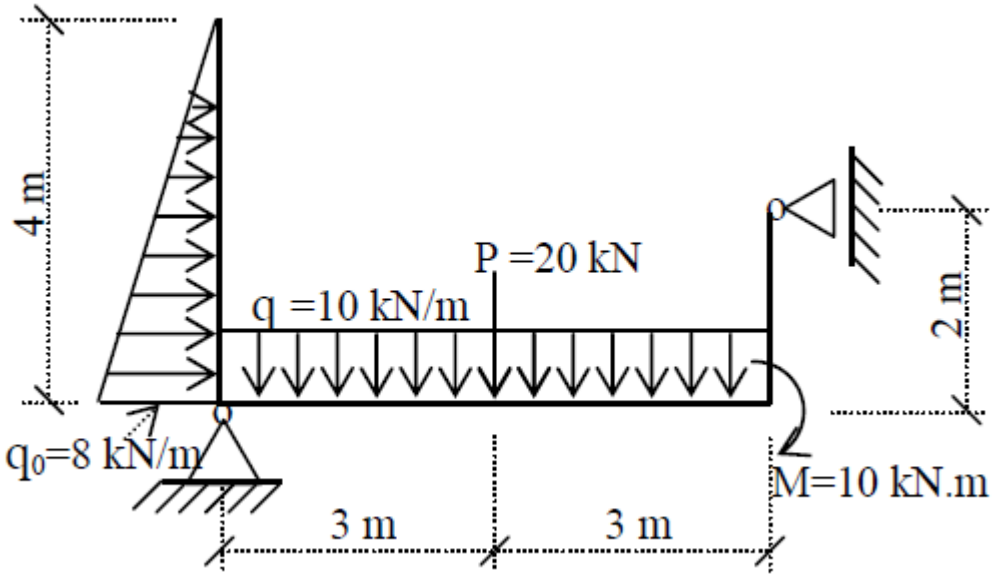


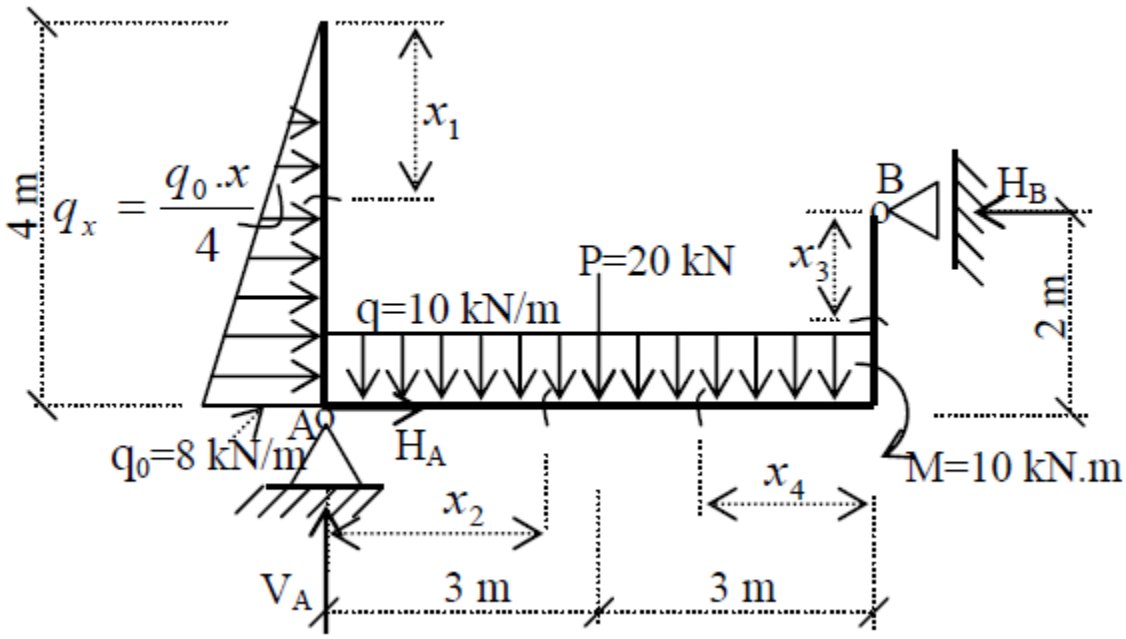
T ve *N* diyagramlarında işaretlenmiş olan mesnet tepkilerinin değerleri dikkate alındığında

$$\sum X = 15.6 - 75 - 15 = 0\,; \quad \sum Y = -22{,}625 + 66{,}625 - 20 - 8.3 = 0$$

olarak dengenin sağlanmış olduğu görülmektedir.

PROBLEM 4.7 Şekildeki çerçevenim M, N. T diyagramlarını çerçeve bölgelerinde kesim yaparak çiziniz

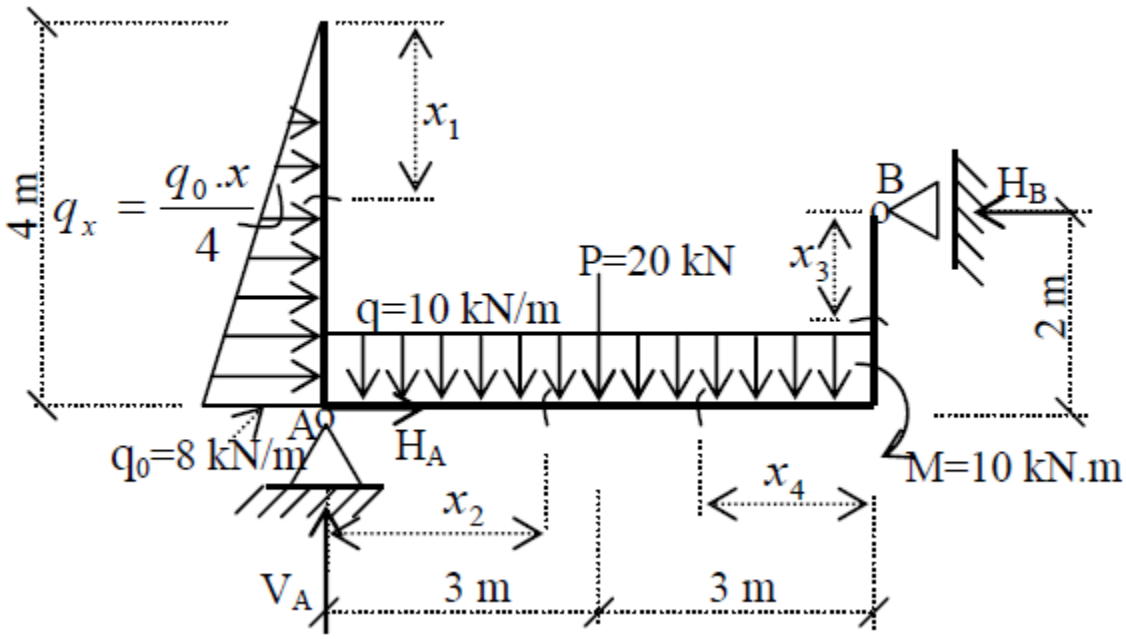




$$\sum Y = 0 \Rightarrow \quad V_A - P - q.6 = 0 \;\Rightarrow\; V_A = 20 + 10.6 = 80\ kN$$

$$\sum M_A = 0 \Rightarrow \quad -H_B.2 + M + q.6.3 + P.3 + \frac{q_9.4}{2}.\frac{1}{3}.4 = 0$$

$$H_B = \frac{10 + 10.6.3 + 20.3 + \frac{8.4}{2}.\frac{1}{3}.4}{2} = 135{,}6667\ kN$$

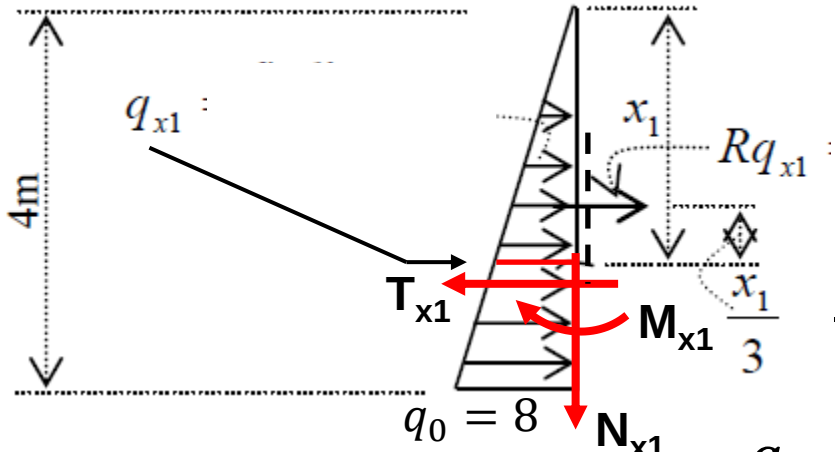


$$\sum M_B = -\frac{q_0.4}{2}.(\frac{2.4}{3}-2)+V_A.6-q.6.3-P.3+M-H_A.2=0$$

$$H_A = \frac{-\frac{8.4}{2}.(0,66667)+80.6-10.6.3-20.3+10}{2} = 119,6667\,kN$$

$$\sum X = \frac{q_0.4}{2}+H_A-H_B=0 \Rightarrow \frac{8.4}{2}+119,6667-135,6667=0$$

Çerçevenin bölgeleri için M, T ve N'nin değerlerinin hesaplanması



$$\frac{q_o}{4} = \frac{q_{x1}}{x_1} \rightarrow q_{x1} = \frac{q_o x_1}{4}$$

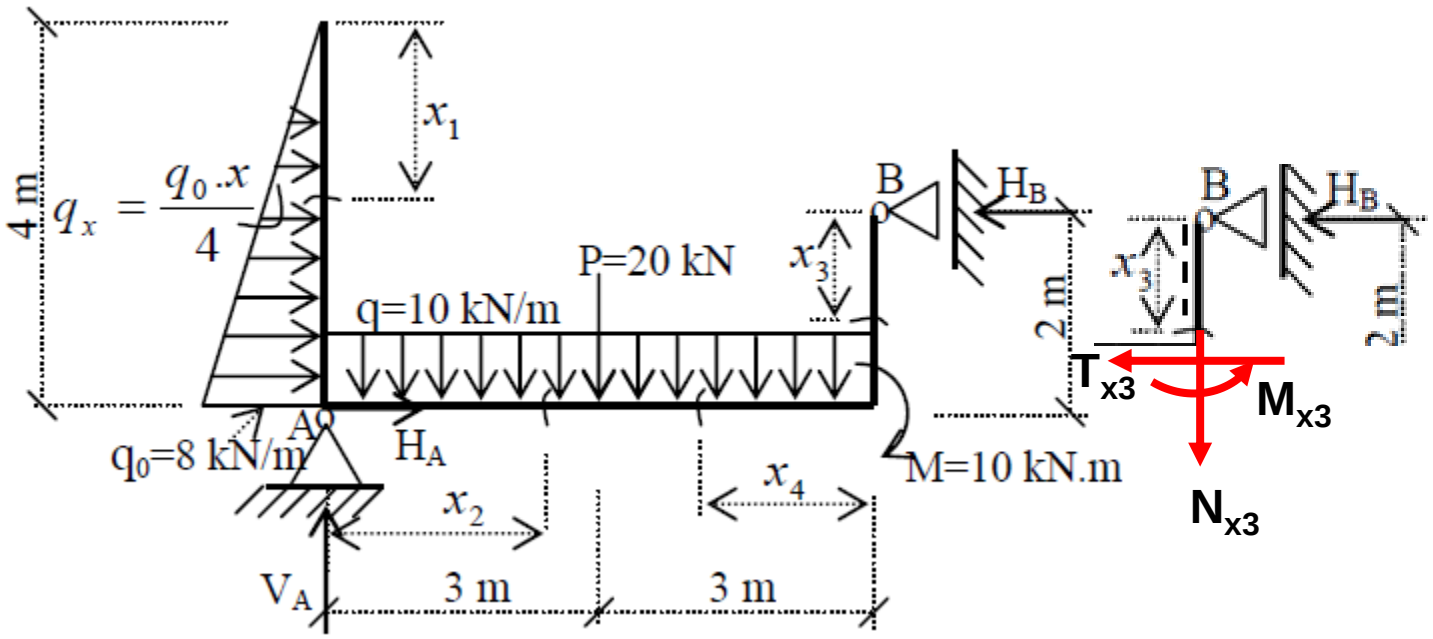
$$R_{q_{x1}} = \frac{q_{x1} x_1}{2} = \frac{q_o x_1}{4}\frac{x_1}{2} = \frac{8{x_1}^2}{8} = {x_1}^2$$

$0 \le x_1 \le 4$

$$M_{x1} = -R\,q_{x1}.\frac{x_1}{3} = -x_1^2.\frac{x_1}{3} = -\frac{x_1^3}{3}\,; \quad T_{x1} = R\,q_{x1} = x_1^2\,; \quad N = 0.$$

$$x_1 = 0 \Rightarrow M_{x1} = 0\,;\ T_{x1} = 0\,;\ x_1 = 2\,m \Rightarrow M_{x1} = -\frac{2^3}{2} = -2{,}666\,kN.m\,;\ T_{x1} = 4\,kN.$$

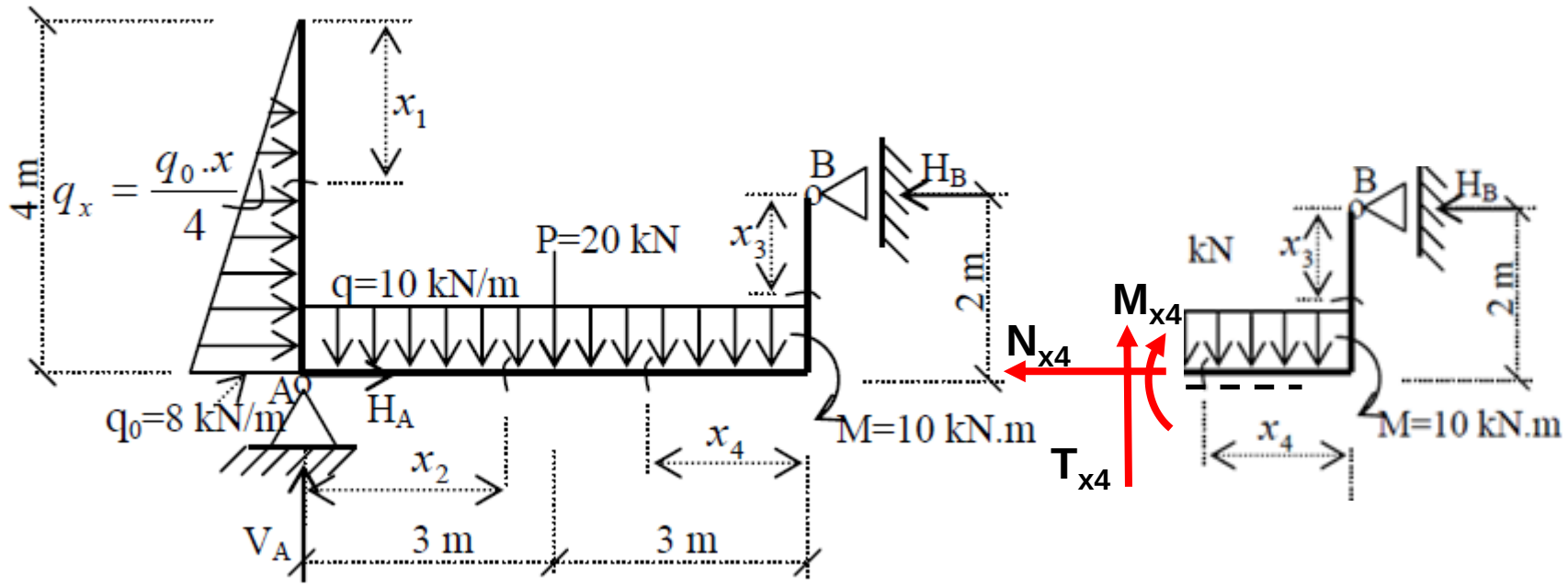
$$x_1 = 4\,m \Rightarrow M_{x1} = -\frac{4^3}{3} = -21{,}333\,kN.m \quad T_{x1} = 16\,kN.$$



$0 \le x_3 \le 2$

$M_{x3} = H_B.x_3 \; ; \quad T = -H_B = -135.6667\,kN \; ; \quad N = 0.$

$x_{x3} = 0 \Rightarrow \quad M_{x3} = 0 \; ; \quad x_3 = 2\,m \Rightarrow \quad M_{x3} = (135{,}6667).2 = 271{,}3333\,kN.m$



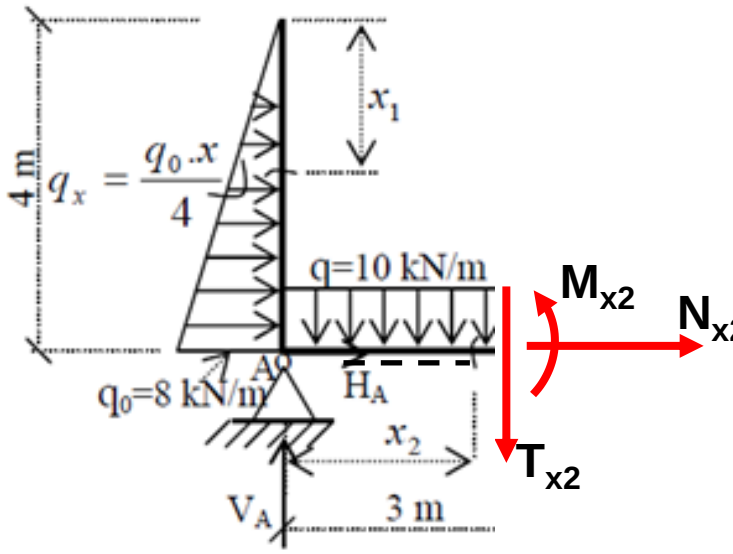
$$0 \le x_4 \le 3$$

$$M_{x4} = H_b.2 - M - q.\frac{x_4^2}{2} = 271{,}333 - 10 - 10.\frac{x_4^2}{2} = 261{,}333 - 5.x_4^2 \ ;$$

$$T = q.x_4 \ ; \quad N = -H_B = -135{,}666\,kN. \ \ x_4 = 0 \Rightarrow M_{x4} = 261{,}333\,kN.m \ ; \quad T_{x4} = 0 \ ;$$

$$x_4 = 1{,}5\,m \Rightarrow \quad M_{x4} = 261{,}3333 - 5.(1{,}5)^2 = 250{,}08333\,kN.m \ ;$$

$$x_4 = 3\,m \Rightarrow \quad M_{x4} = 261{,}333 - 5.(3)^2 = 216{,}333\,kN.m \ ; \quad T_{x4} = 10.3 = 30\,kN.$$



$$0 \le x_2 \le 3$$

$$M_{x2} = \frac{8.4}{2}\frac{1}{3}4 - 10.x_2\frac{x_2}{2} + V_A.x_2$$

$$V_A = 80\ kNm\ H_A = 119.667kN$$

$$x_2 = 0 \rightarrow M_{x2} = 21.33$$

$$x_2 = 3 \rightarrow M_{x2} = 21.33 - 45 + 80.3 = 216.33\ kNm$$

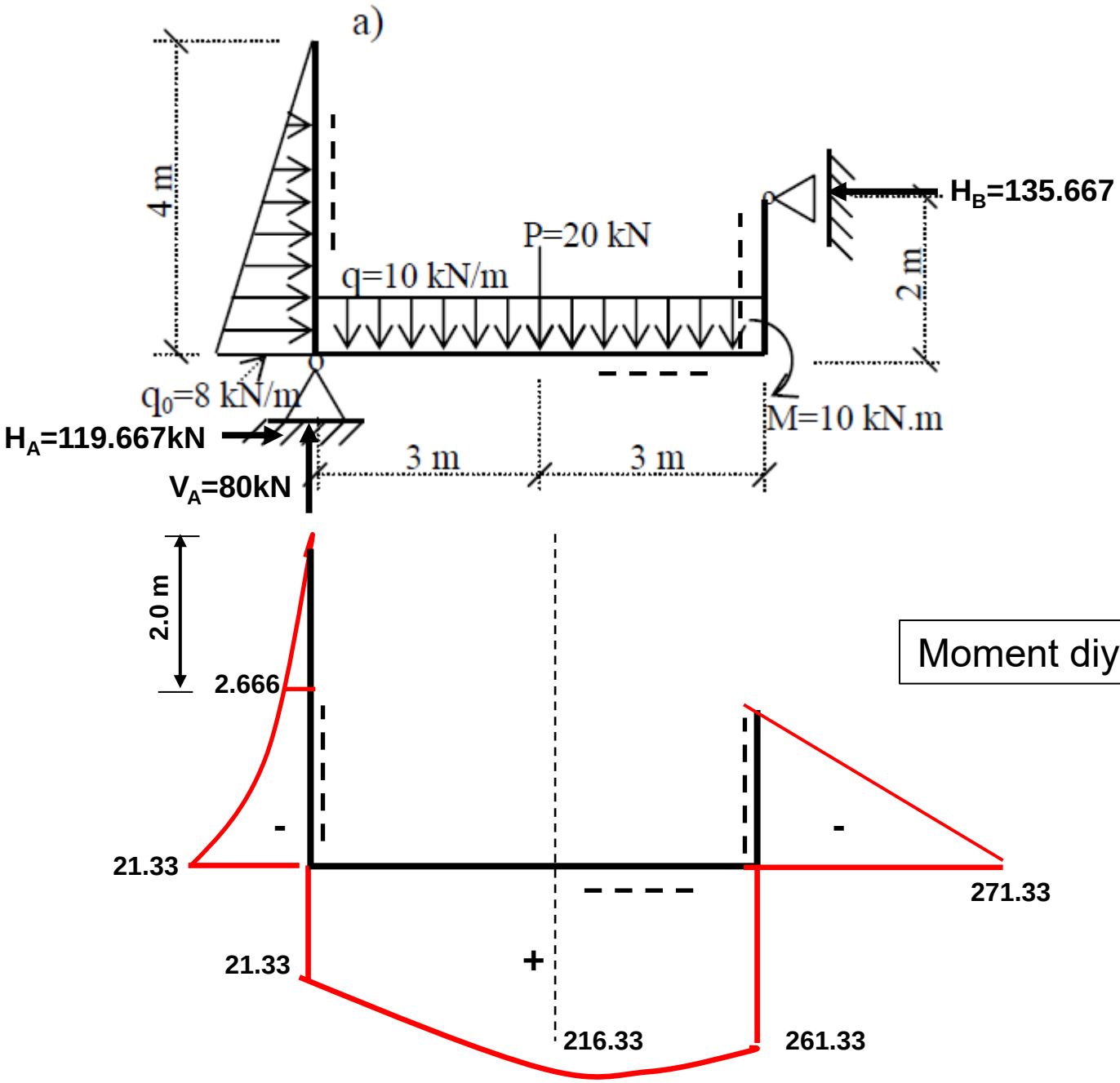
$$T_{x2} = V_A - 10.x_2$$

$$x_2 = 0 \rightarrow T_{x2} = 80\ kN$$

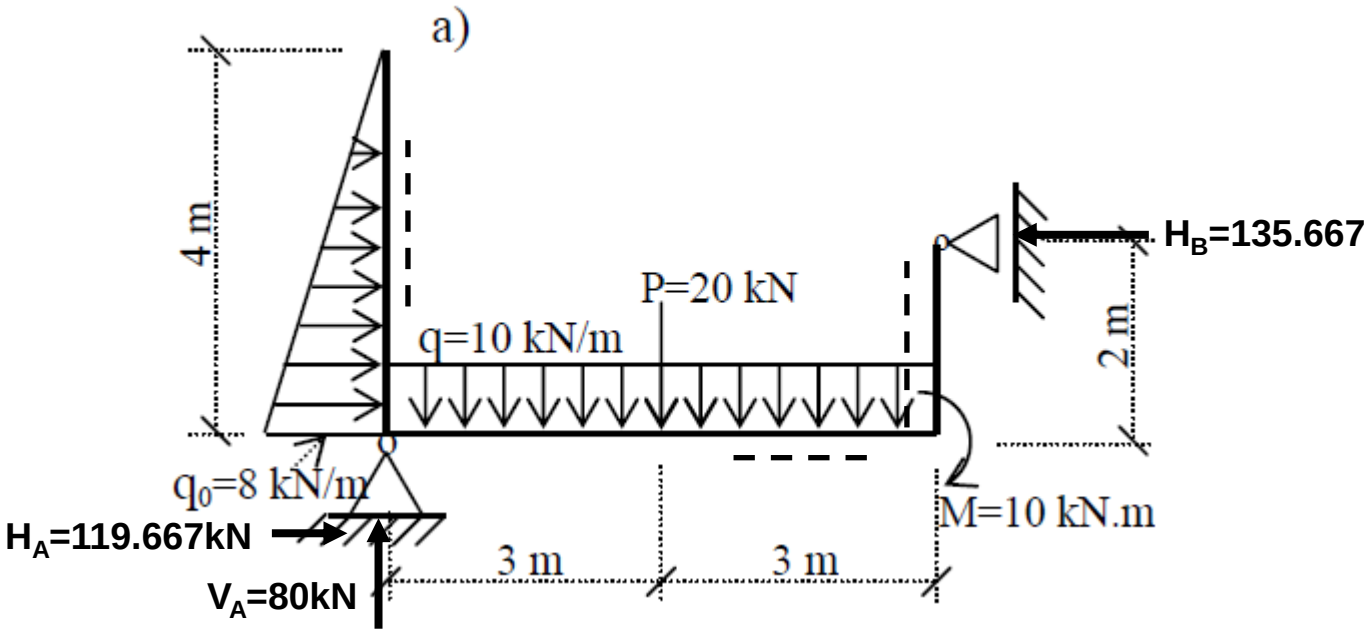
$$x_2 = 3 \rightarrow T_{x2} = 80 - 30 = 50\ kN$$

$$N_{x2} = -H_A - \frac{8.4}{2} = -119.667 - 16 = -135.667kN$$

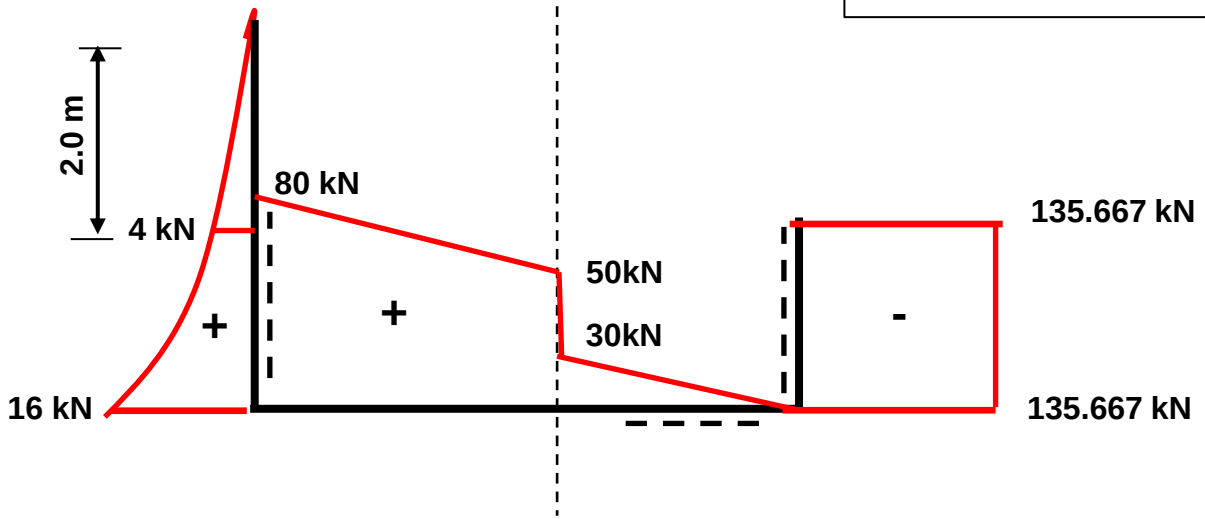
$$x_2 = 0 \rightarrow N_{x2} = -135.667\ kN\ x_2 = 3 \rightarrow N_{x2} = -135.667\ kN$$

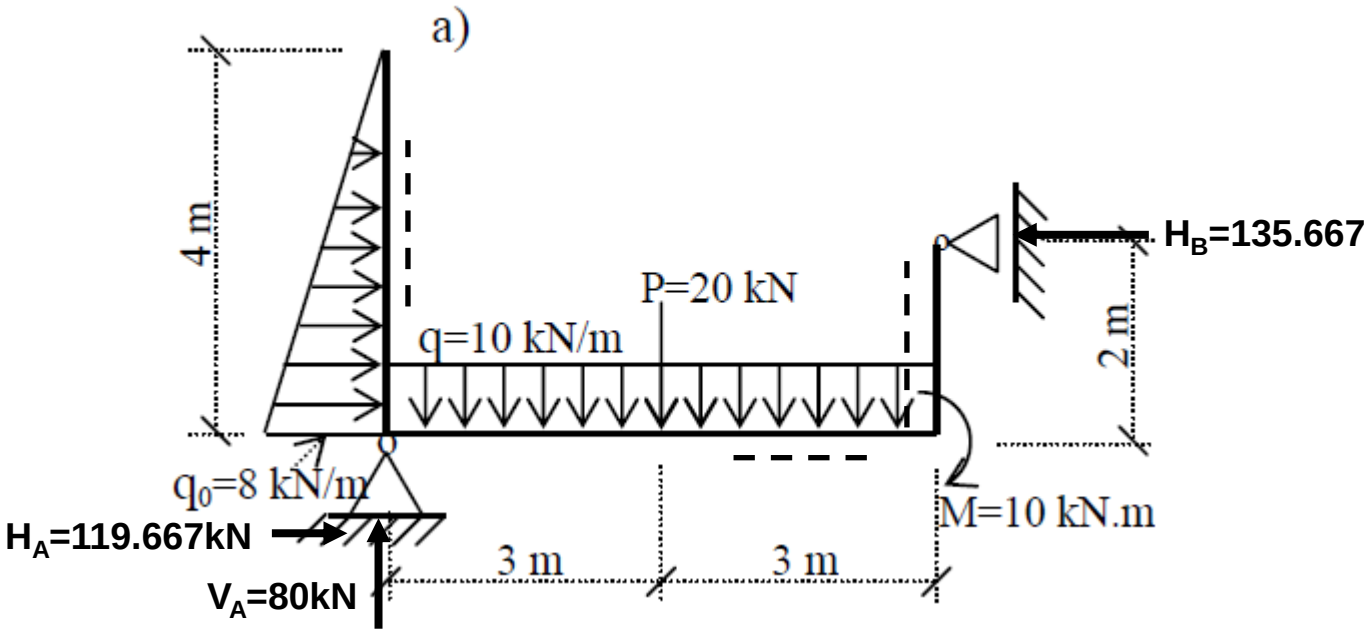


Moment diyagramı

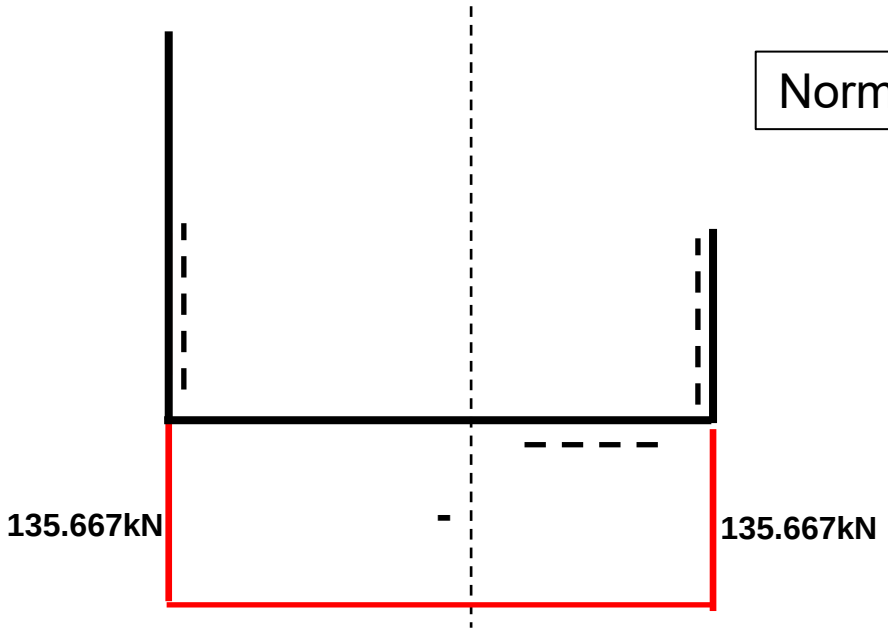


Kesme kuvveti diyagramı

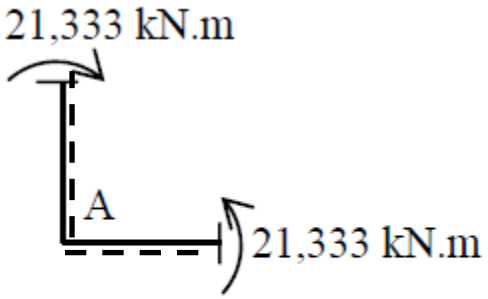


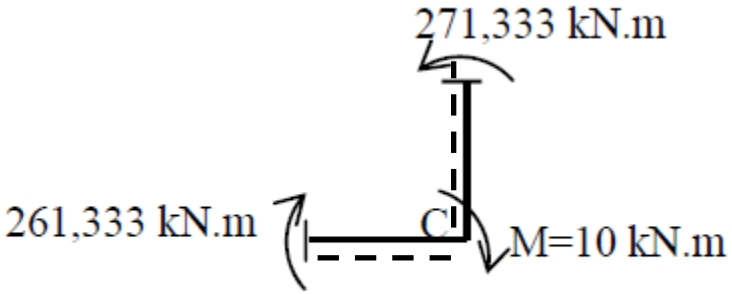


Normal kuvveti diyagramı



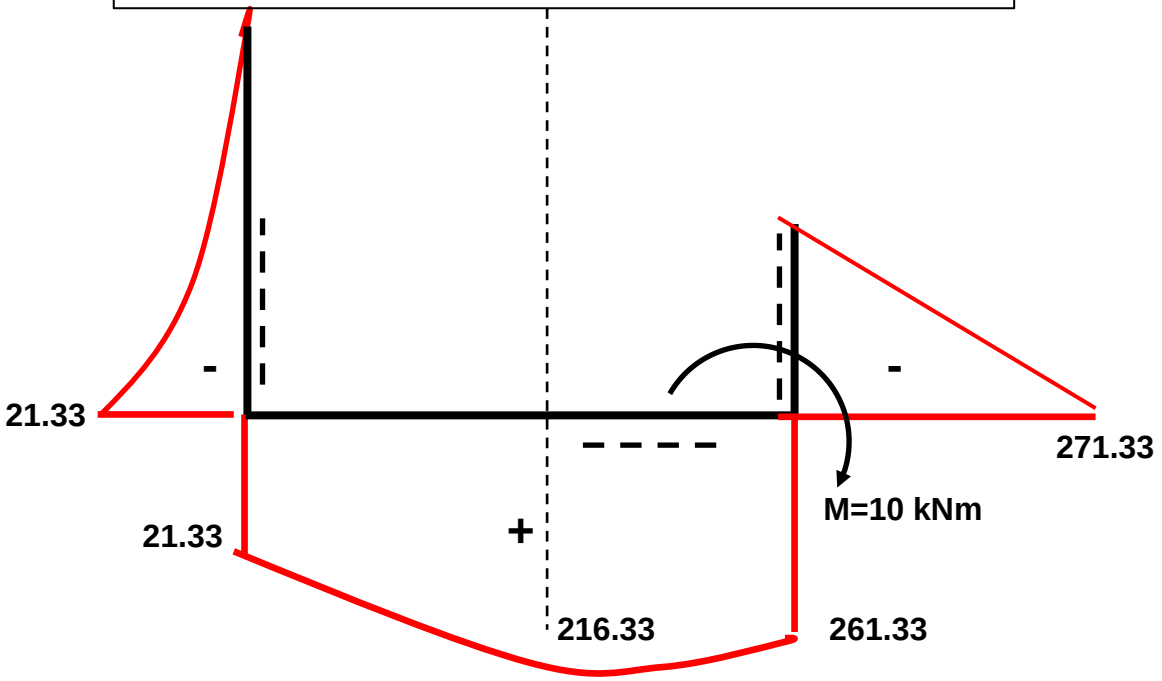
Çerçevenin genel denge kontrolü



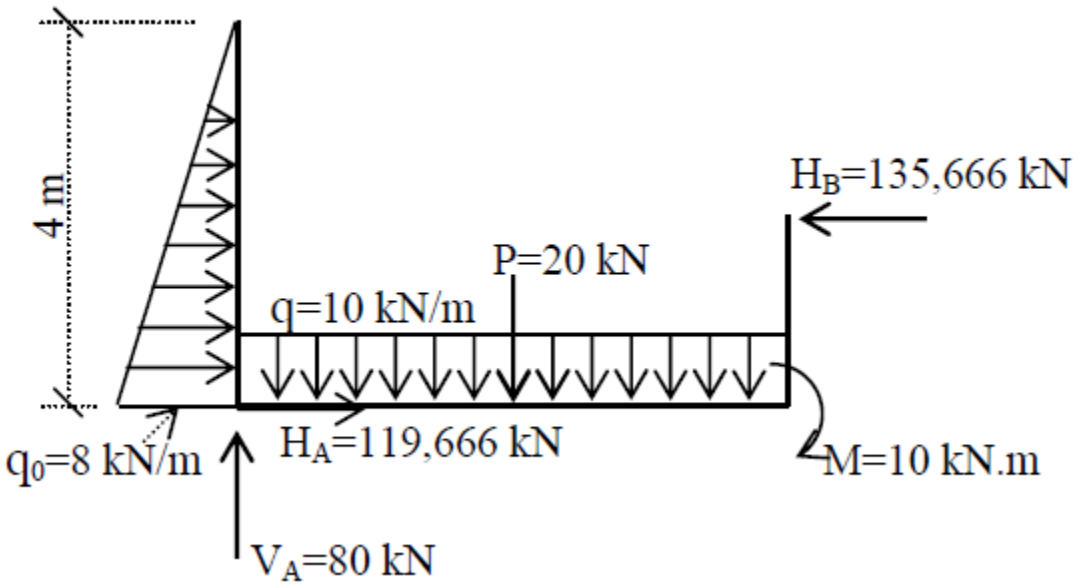


$$\sum M_A = 21{,}333 - 21{,}333 = 0 \qquad \sum M_C = 261{,}333 + 10 - 271{,}333 = 0$$

Düğüm noktası Moment denge kontrolü



Yatay ve düşey kuvvetler denge kontrolü



$$\sum X = H_A + \frac{q_0.4}{2} - H_B = 119{,}666 + \frac{8.4}{2} - 135{,}666 = 0$$

$$\sum Y = V_A - q.6 - P = 80 - 10.6 - 20 = 0$$